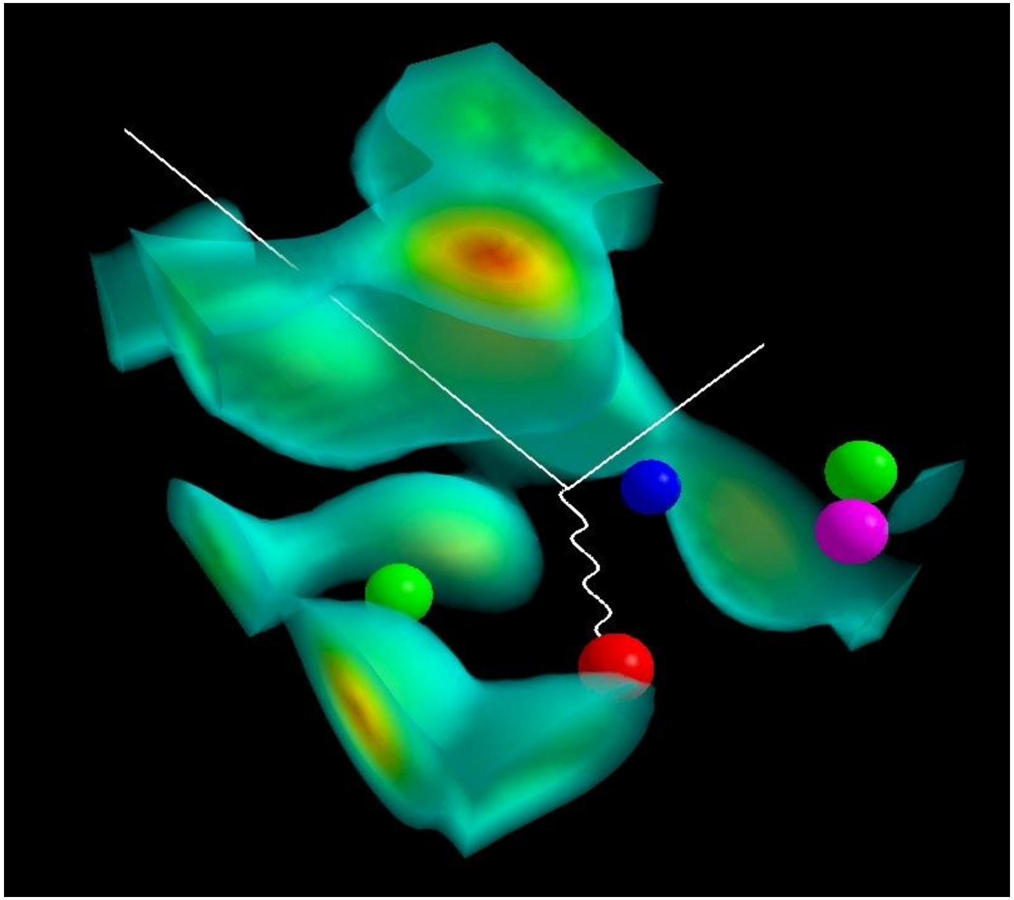
Standard Model Tests and Fragmentation Functions



Australian Government

Australian Research Council

Anthony W. Thomas

4th Workshop on Exclusive Reactions at High Momentum Transfer

Jefferson Lab : May 21st 2010

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

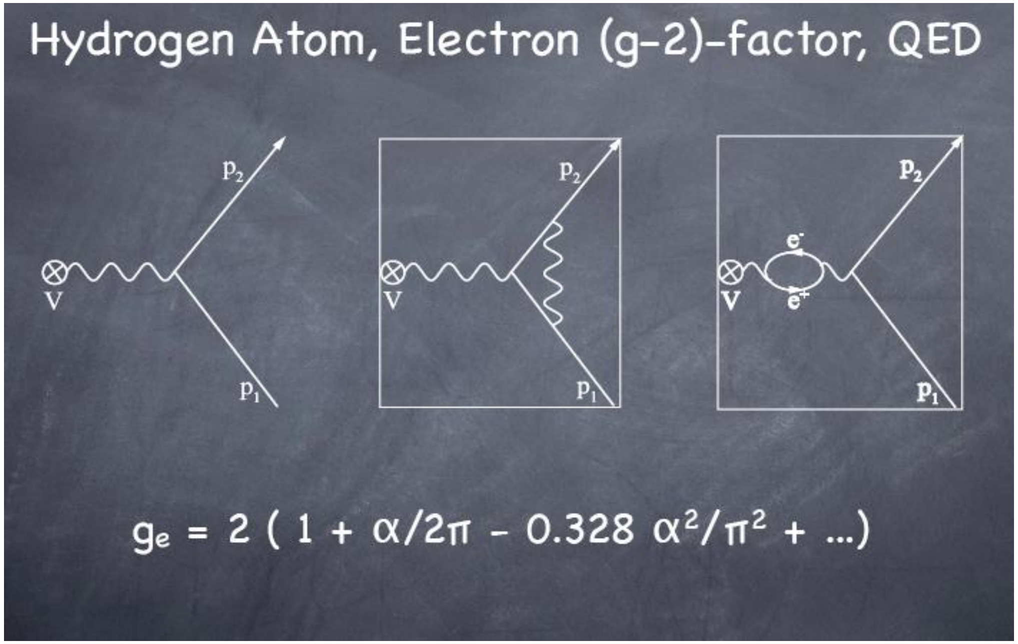
Outline

- Testing the Neutral Current Couplings
- The NuTeV anomaly
- Resolution of the NuTeV anomaly
    - CSV in parton distribution functions
    - a new EMC effect
- CSV at an EIC
- Fragmentation functions

Non-perturbative QCD

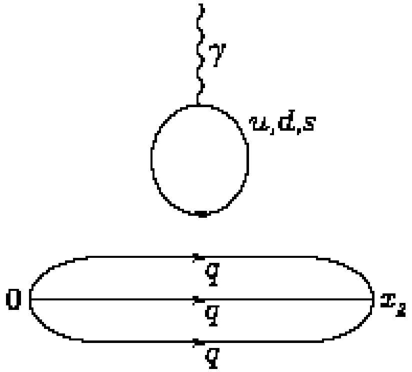
Testing Non-Perturbative QCD

- Strangeness contribution is a vacuum polarization effect, analogous to Lamb shift in QED



Hydrogen Atom, Electron (g-2)-factor, QED

$g_e = 2\,(1 + \alpha/2\pi - 0.328\,\alpha^2/\pi^2 + \ldots)$



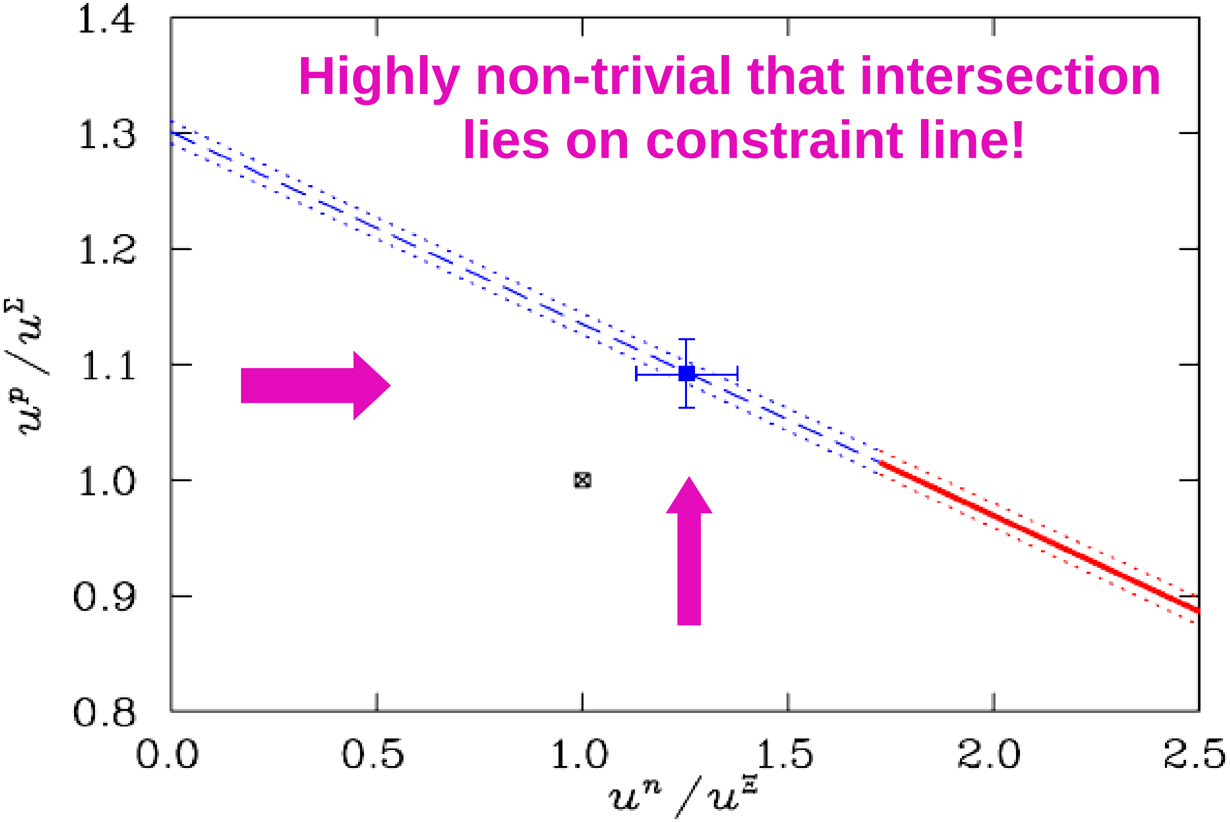
- It is a fundamental test of non-perturbative QCD

Strange Quarks in the Proton

There have been a number of major steps forward recently, both theory and experiment :

- Calculation of $G_{E,M}^{s}$ (Q^2) :
  - Direct: Kentucky (χQCD : K.-F. Liu)
  - Indirect: JLab-Adelaide
- Experimental determination of $G_{E,M}^{s}$ (Q^2)
  - G0 and Happex
  - Mainz PVA4 (arXiv:0903.2733) and Bates
- Agreement between theory and experiment excellent
  - consistent global analysis valuable

First Accurate Determination of G_M^s from QCD



Highly non-trivial that intersection lies on constraint line!

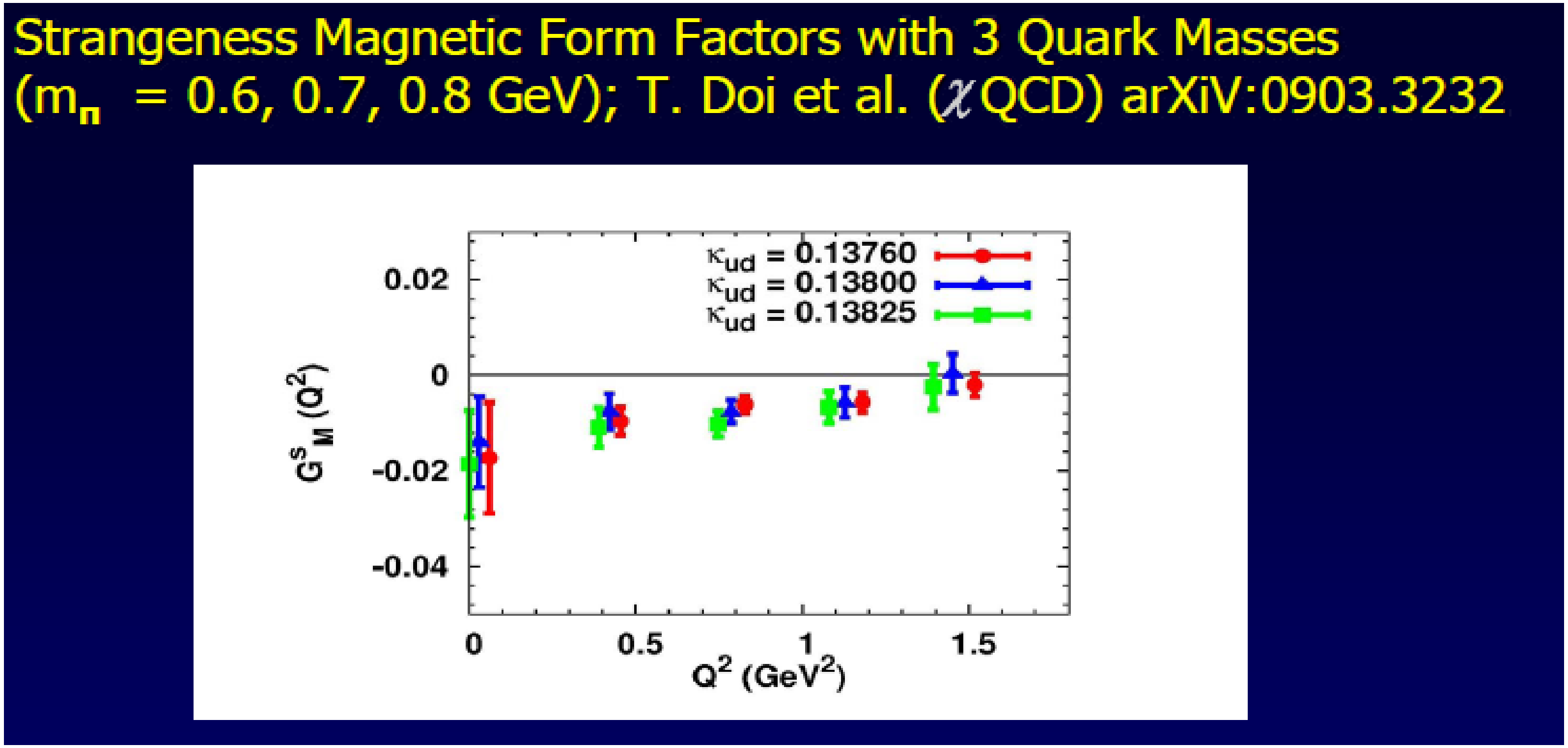
1.10±0.03

1.25±0.12

Yields :$G_M^s = -0.046 \pm 0.019\ \mu_N$

Leinweber et al., PRL 94 (2005) 212001

Direct Calculation of $G_M^s(Q^2)$ – K.-F. Liu et al.



Strangeness Magnetic Form Factors with 3 Quark Masses (m_π = 0.6, 0.7, 0.8 GeV); T. Doi et al. (χQCD) arXiV:0903.3232

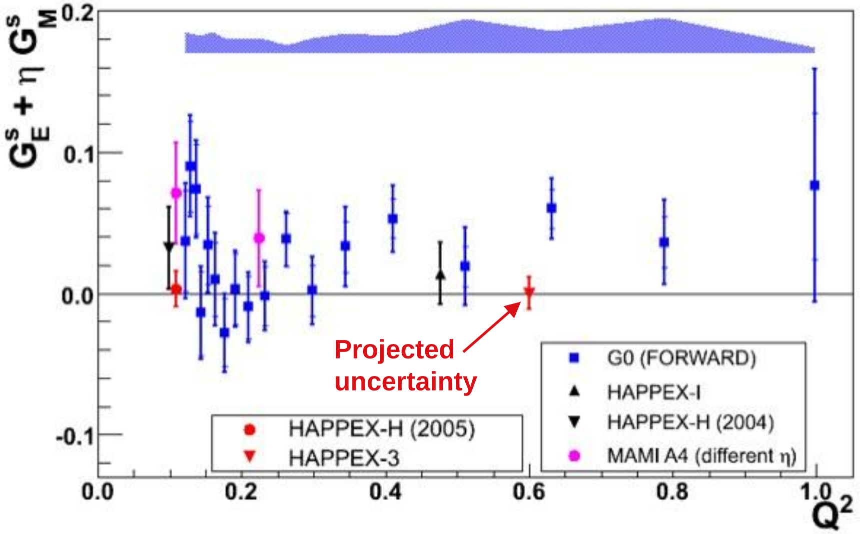
$$G_M^S(Q^2=0) = -\,0.017(25)(07)\ \mu_N$$

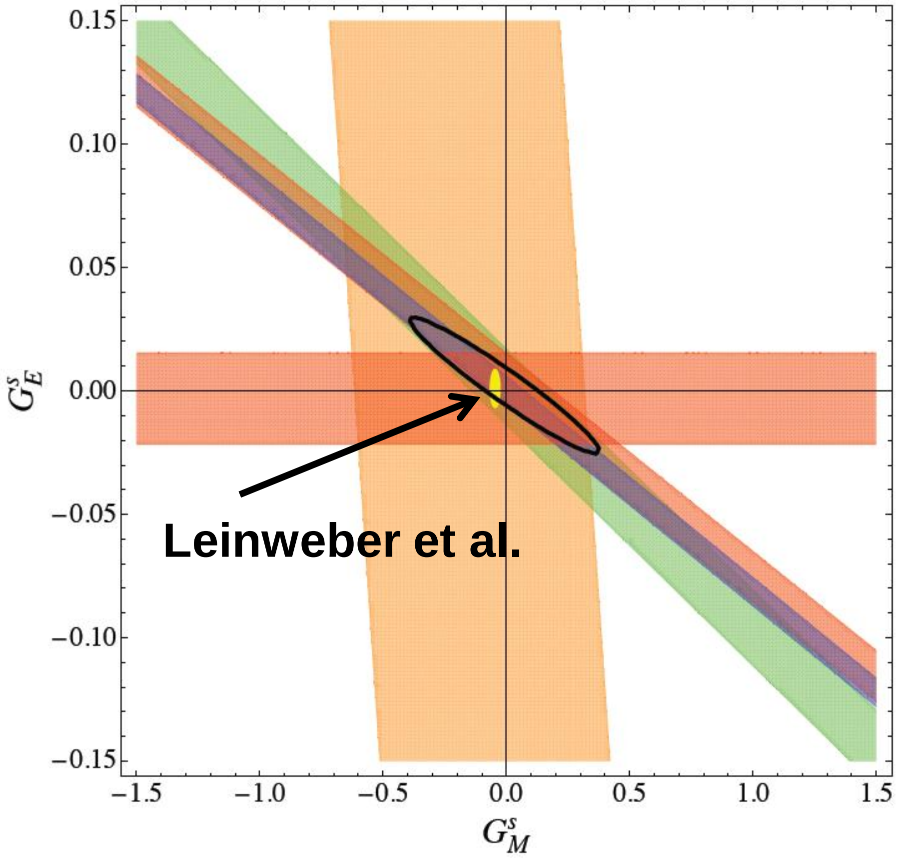
c.f. -0.046 ± 0.019 (Leinweber et al.)

N.B. Result of Doi et al. would increase by factor ~1.8 when light quark mass takes physical value with m_s fixed (Wang et al., hep-ph/0701082 :Phys Rev D75, (2008))

Global Analysis of PVES Data

$Q^2 = 0.1$ GeV^2


Projected uncertainty


Leinweber et al.

- ➢**Proton not all that strange**

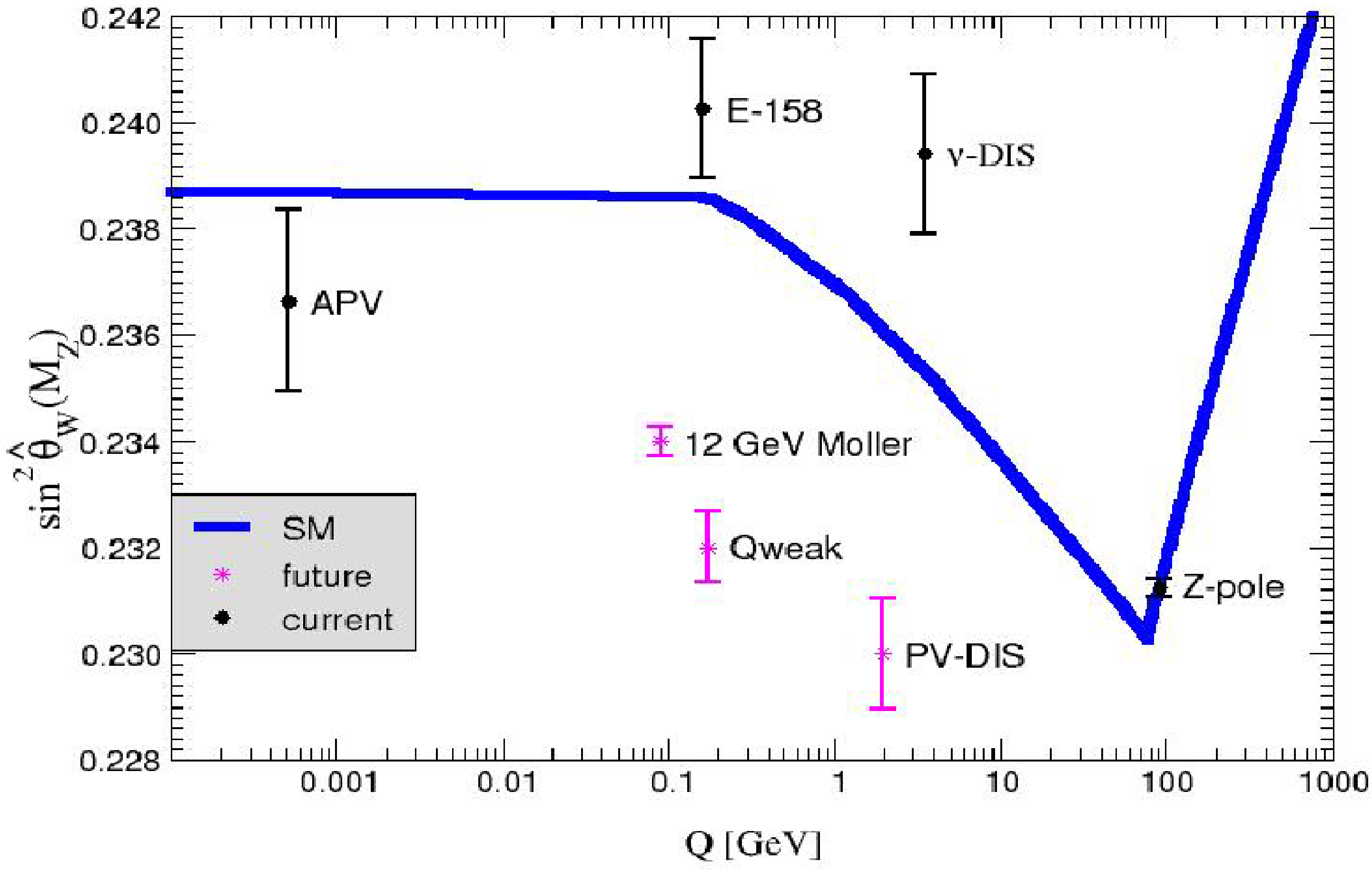
- ➢**New data not yet included at 0.23 and 0.6 GeV^2 (PVA4 , G0, HAPPEx III – data taken this year)**

Global analysis: Young et al., PRL 99 (2007)122003 and Young arXiv 1004.5163 [nucl-th]

The Weak Neutral Current

Radiative Corrections Test of Weak Neutral Current

18 months ago….



SM line: Erler & Ramsey-Musolf, Phys.Rev.D72:073003,2005

Success of Strangeness Search Leads Naturally to Measurement of $\sin^2\theta_W$ Using PVES

- Proton target

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{\pi\alpha\sqrt{2}}\right] \frac{\varepsilon G_E^{p\gamma} G_E^{pZ} + \tau G_M^{p\gamma} G_M^{pZ} - \frac{1}{2}(1 - 4\sin^2\theta_W)\varepsilon' G_M^{p\gamma} \tilde{G}_A^p}{\varepsilon (G_E^{p\gamma})^2 + \tau (G_M^{p\gamma})^2}$$

Neutral-weak form factors

Axial form factor

Assume charge symmetry:

$$4G_{E,M}^{pZ} = \underline{(1 - 4\sin^2\theta_W)} G_{E,M}^{p\gamma} - G_{E,M}^{n\gamma} - \underline{G_{E,M}^{s}}$$

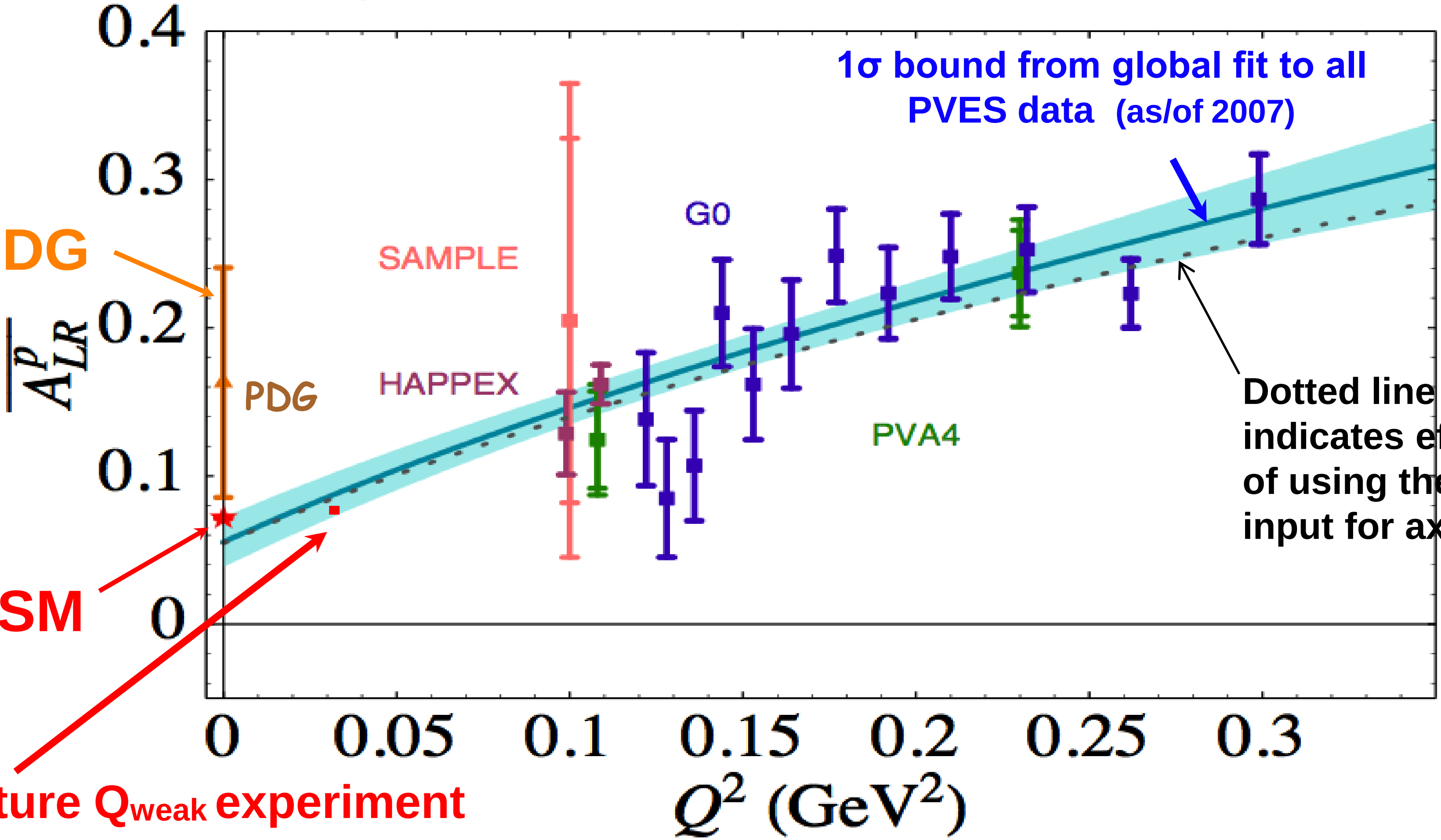
Proton weak charge
(tree level)

Strangeness

Use data to constrain the parameters of the electroweak theory

Use Global Fit to Extract Slope at 0º and $Q^2 = 0$

$$\overline{A^p_{LR}} = A_z/(-G_F Q^2/4\pi\alpha\sqrt{2}) = Q^p_{weak} + Q^2 B(Q^2)$$



1σ bound from global fit to all PVES data (as/of 2007)

Dotted line indicates effect of using theoretical input for axial terms

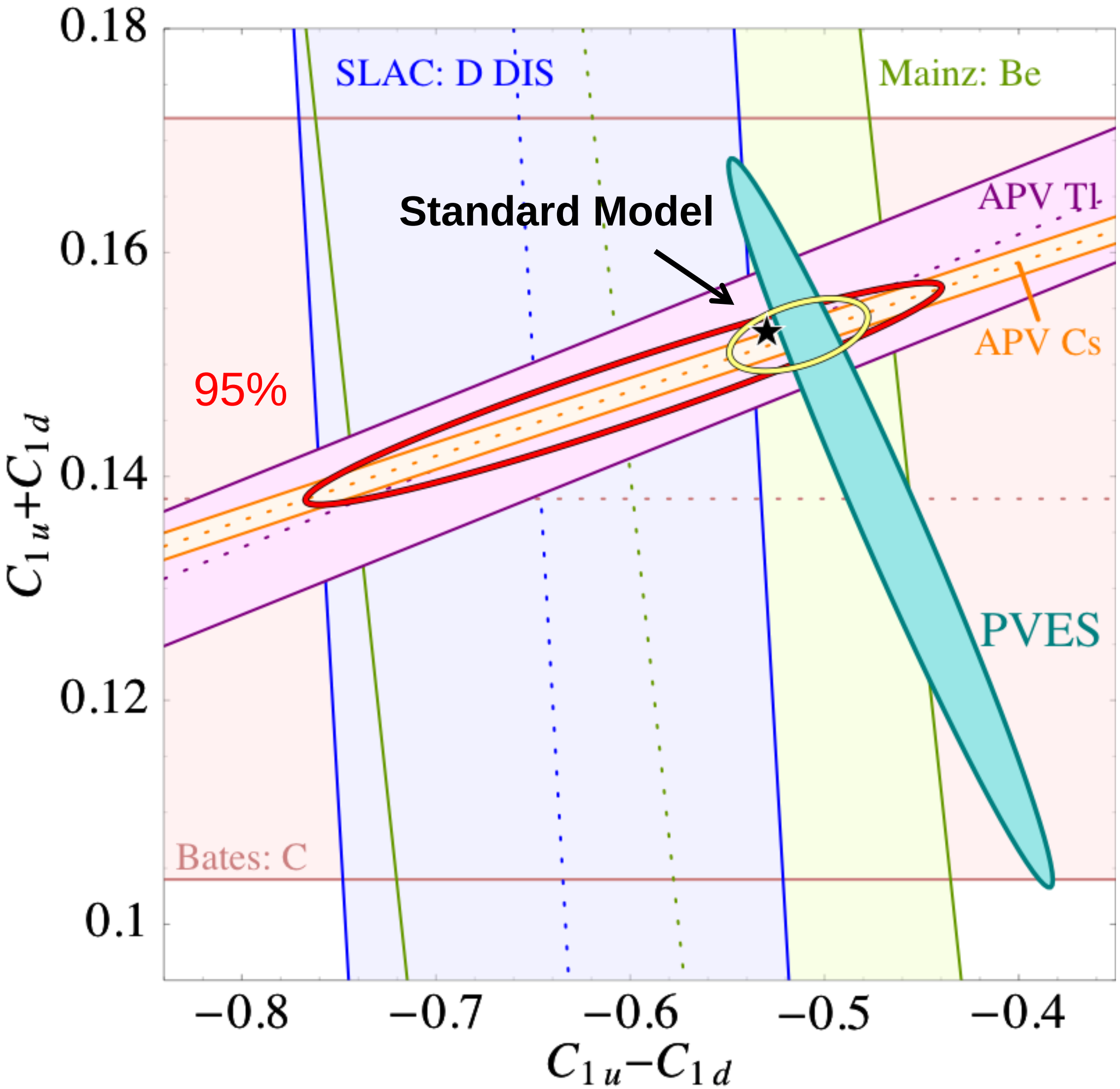
PDG

SM

Future Q_{weak} experiment

(R.D. Young et al., PRL 99, 122003 (2007))

Major progress on C_{1q} couplings



$$Q_{weak} = 2C_{1u} + C_{1d}$$

$$L_{eff} \sim C_{1q}\, \bar{e}\gamma^{\mu}\gamma_5 e\, \bar{q}\gamma_{\mu} q$$

Dramatic improvement in knowledge of weak couplings!

Factor of 5 increase in precision of Standard Model test

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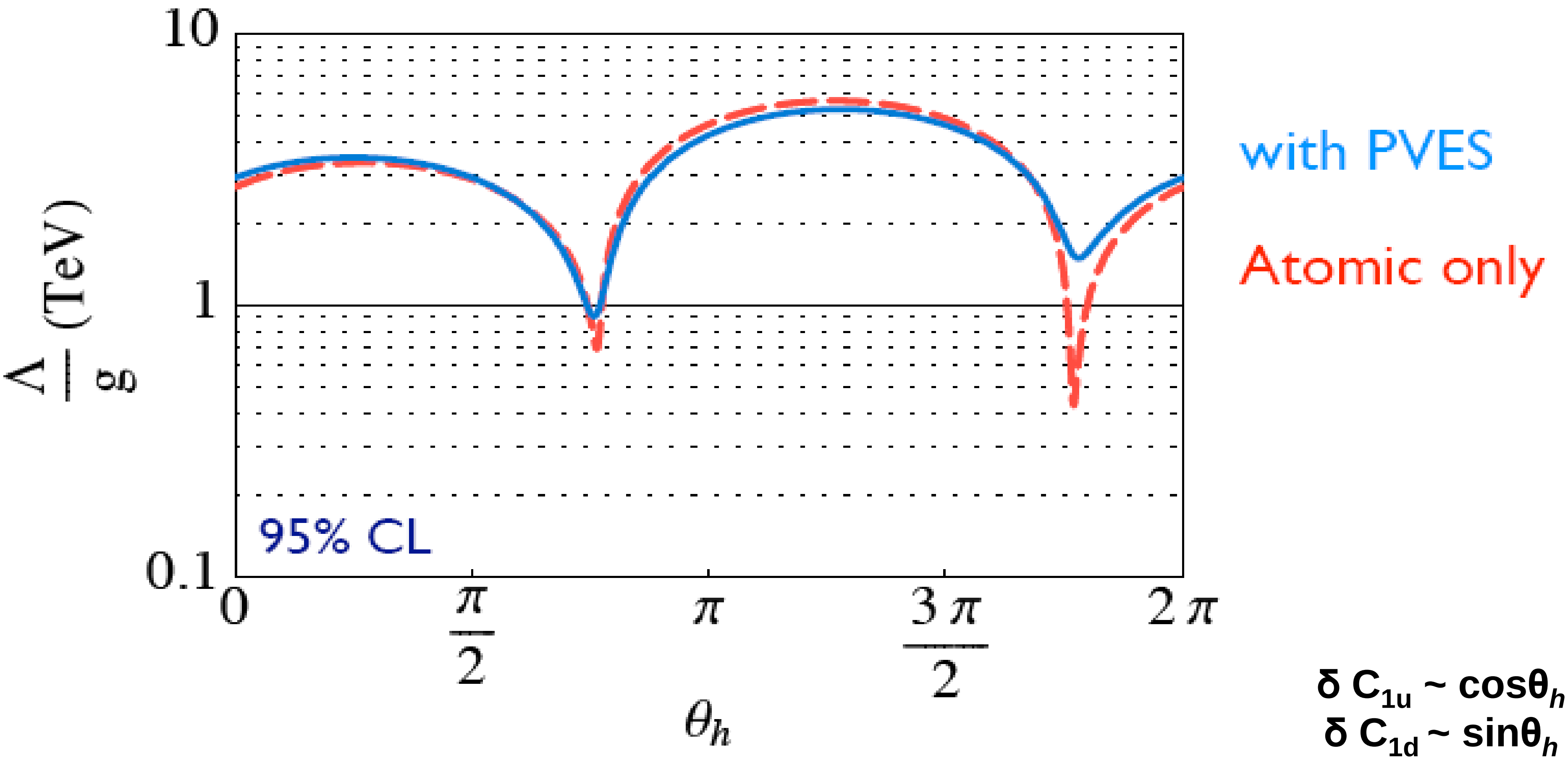
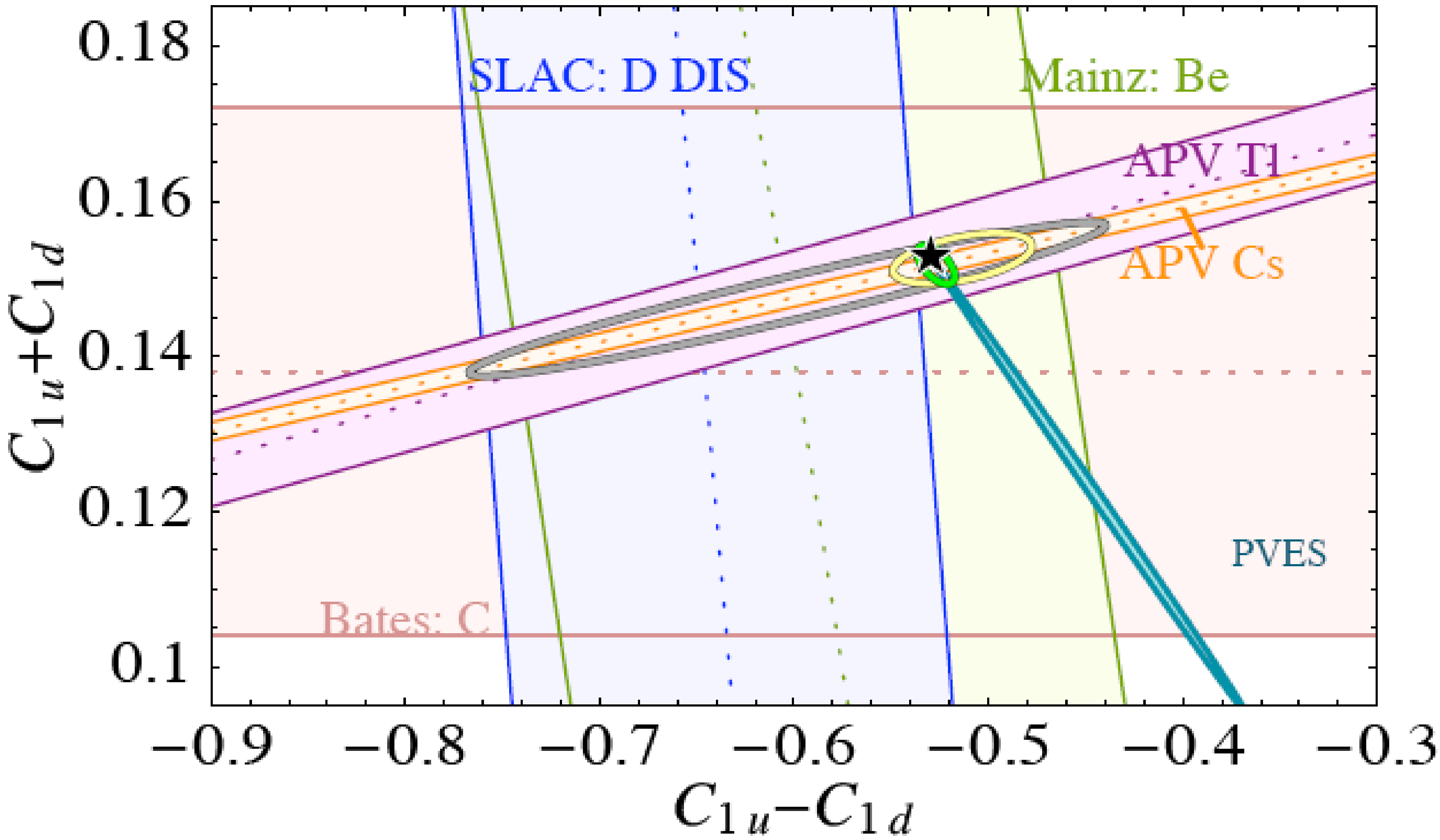
Raises Mass of New Z’ to 0.9 TeV – from 0.4 TeV



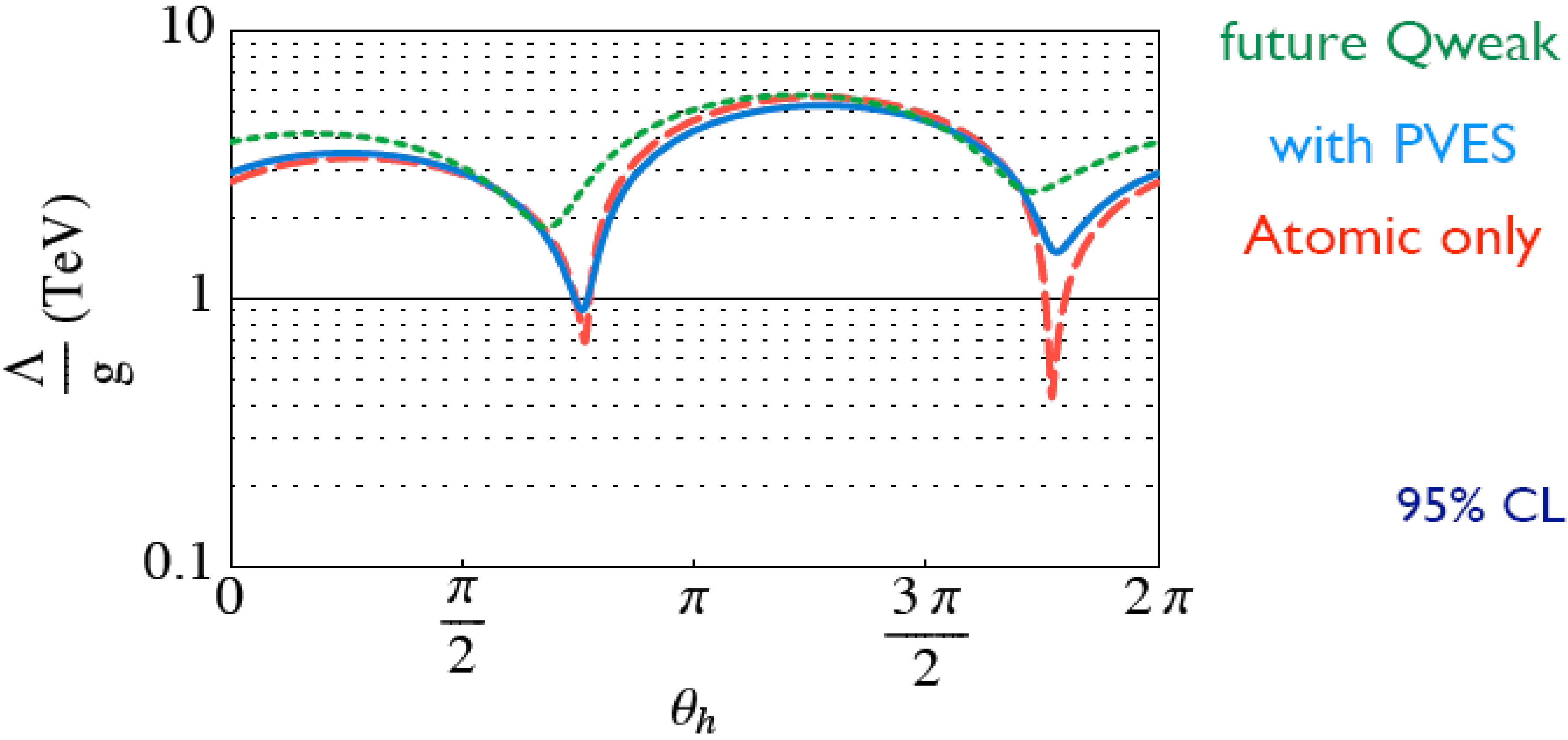
$\delta C_{1u} \sim \cos\theta_h$
$\delta C_{1d} \sim \sin\theta_h$

New physics scale >0.9 TeV! (from 0.4 TeV)

Future Q_{weak} at JLab – <u>if</u> in Agreement with SM



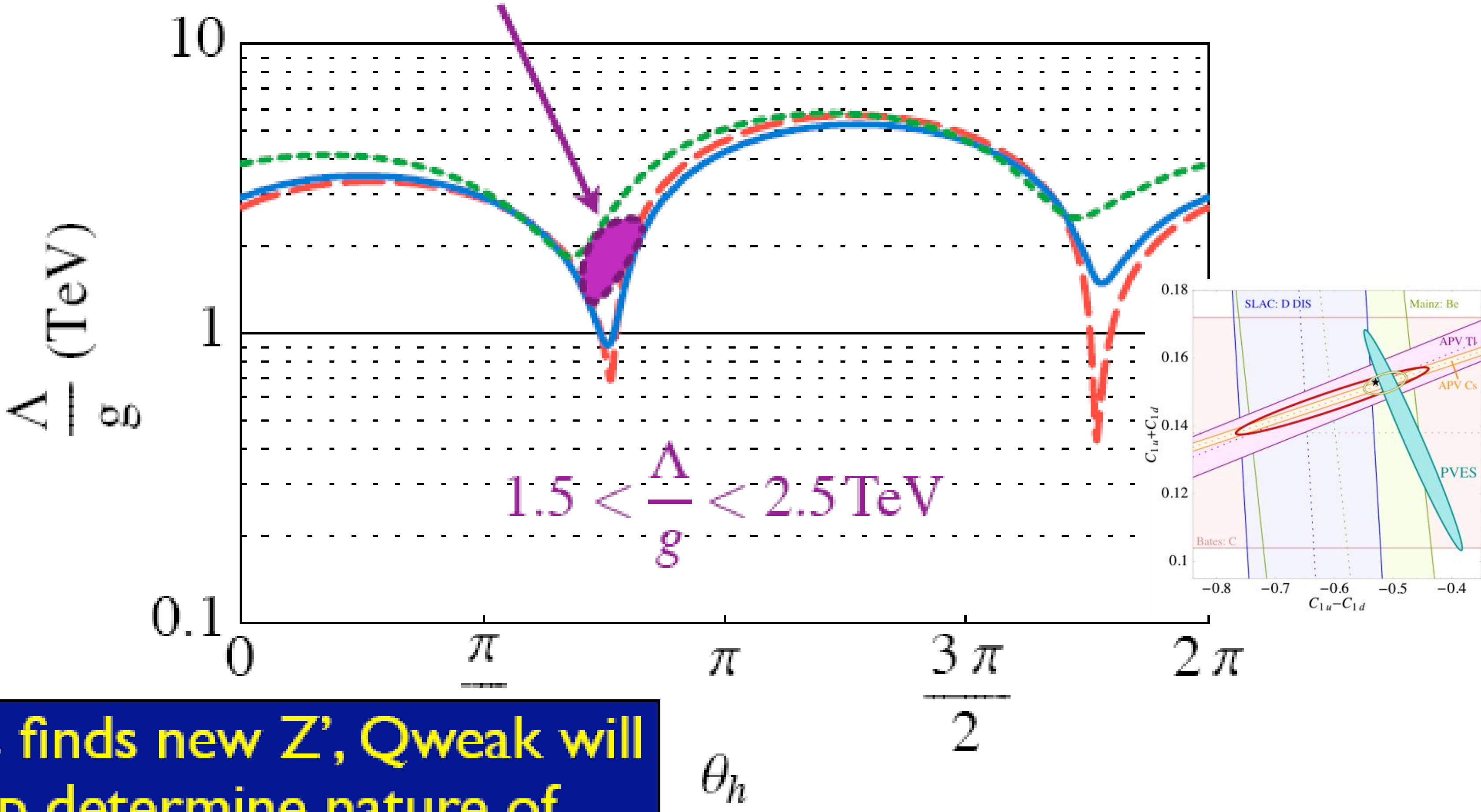
IF in accord with Standard Model…



Qweak constrains new physics to beyond 2 TeV

Or… Discovery

Assume Qweak takes central value of current measurements



$$1.5 < \frac{\Lambda}{g} < 2.5\,\text{TeV}$$

If LHC finds new Z', Qweak will help determine nature of interaction

New Development in Radiative Corrections

- **Initial work, primarily aimed at parity violation in atoms, by Marciano, Sirlin, Erler, Ramsey-Musolf....**

- **In 2009: Gorchtein and Horowitz realized (PRL 102 (2009) 091806) that one of the well studied radiative corrections, the γ-Z box diagram, introduced a strong energy dependence**

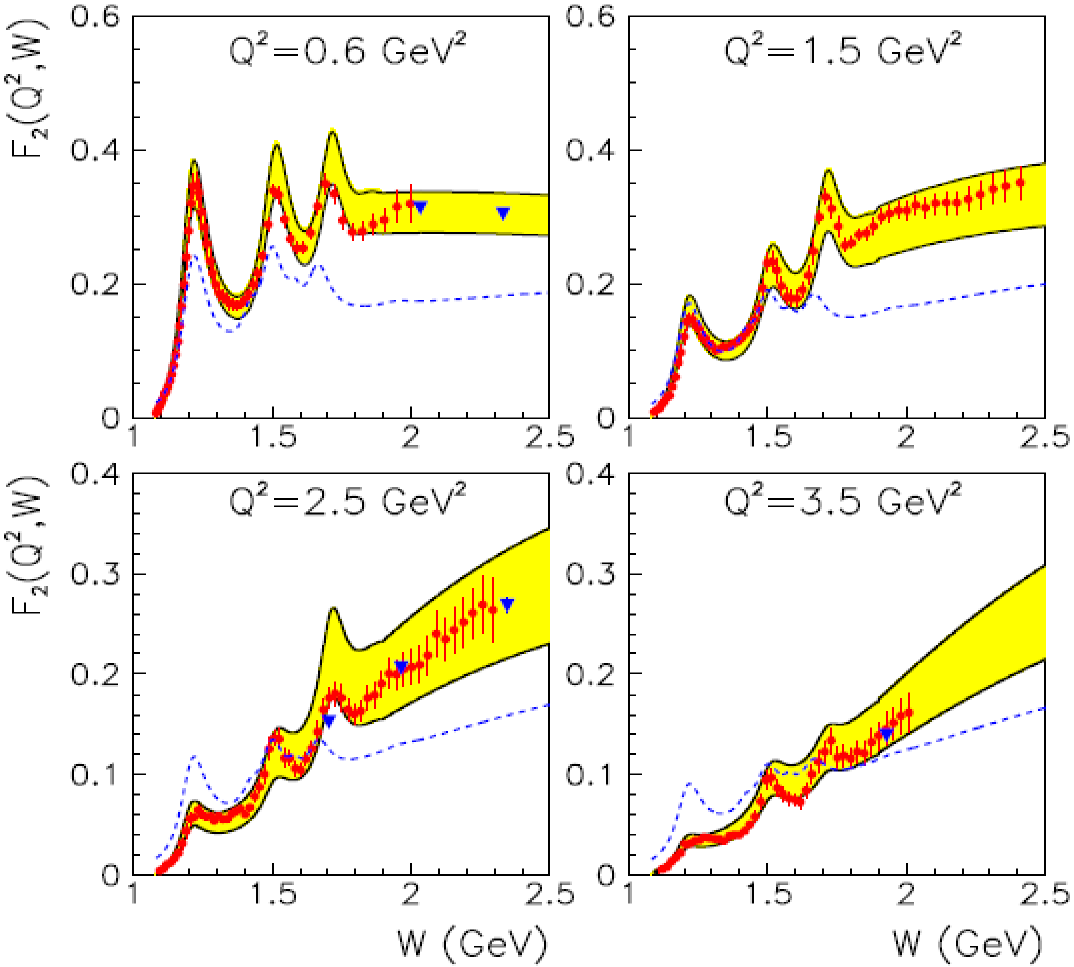
- **That is: a term of order E_e/M_p , which is negligible in atoms, is important at Jlab energies.**

$$A^{\rm PV} \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \rightarrow \frac{G_F}{4\pi\alpha\sqrt{2}}\, t\, Q_W^p$$

$$\begin{aligned} Q_W^p \;&=\; (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') \\ &\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0) \end{aligned}$$

γ-Z Box Diagram

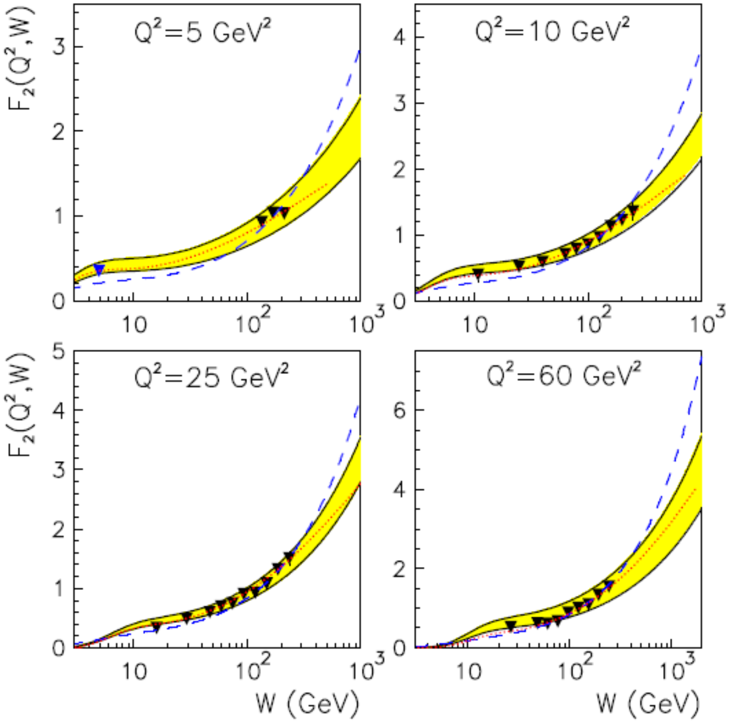
- **Re-examined by Sibirtsev , Melnitchouk, Blunden & Thomas (arXiv:1002.0740 [hep-ph])**
- **Took advantage of CLAS data on photo-production (and HERA data)**



γ- Z Box Diagram (cont.)

- **Use dispersion relation :** $\Re e\,\Box^V_{\gamma Z}(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} dE' \frac{\Im m\,\Box^V_{\gamma Z}(E')}{E' - E}$

$$\Im m\,\Box^V_{\gamma Z}(E) = \frac{\alpha}{(s-M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\rm max}} \frac{dQ^2}{1+Q^2/M_Z^2} \times \left[F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\left(Q^2_{\rm max} - Q^2\right)}{Q^2\left(W^2 - M^2 + Q^2\right)}\right],$$

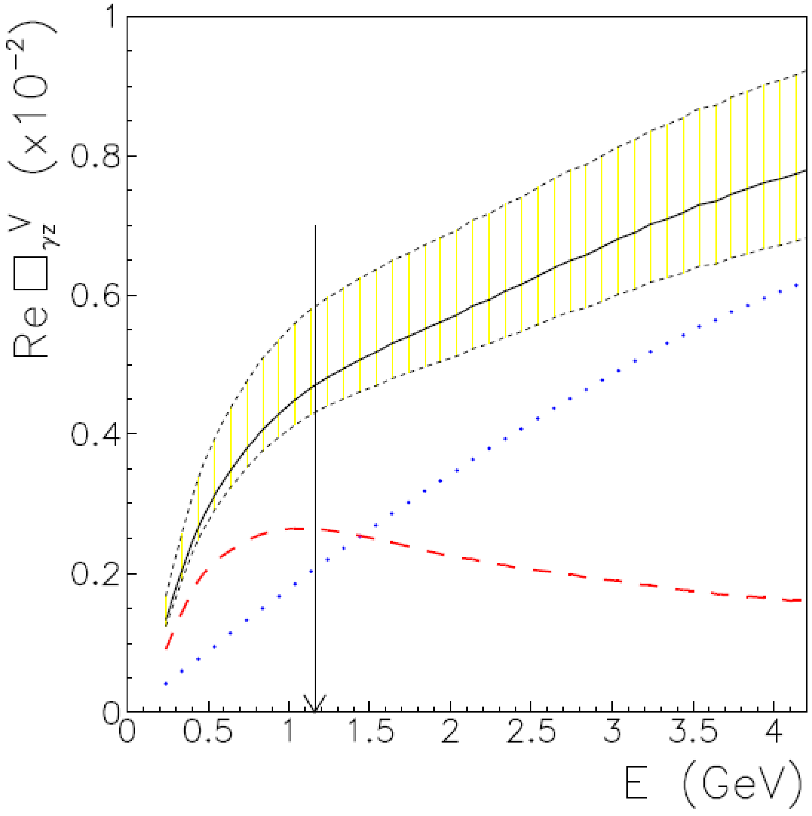


- **With fit to Jlab and HERA data to evaluate the box diagram**

Result for γ – Z box

- **From measurement of A_{PV} at 1.165 GeV (Q_{weak}) the value of Q_W^p extracted needs to be reduced by $0.0047^{+0.0011}_{-0.0004}$ before comparison with the value deduced from atomic PV**

- **This differs from GH correction of ≈ 0.003, because of factor of 2, use of modern data etc.**

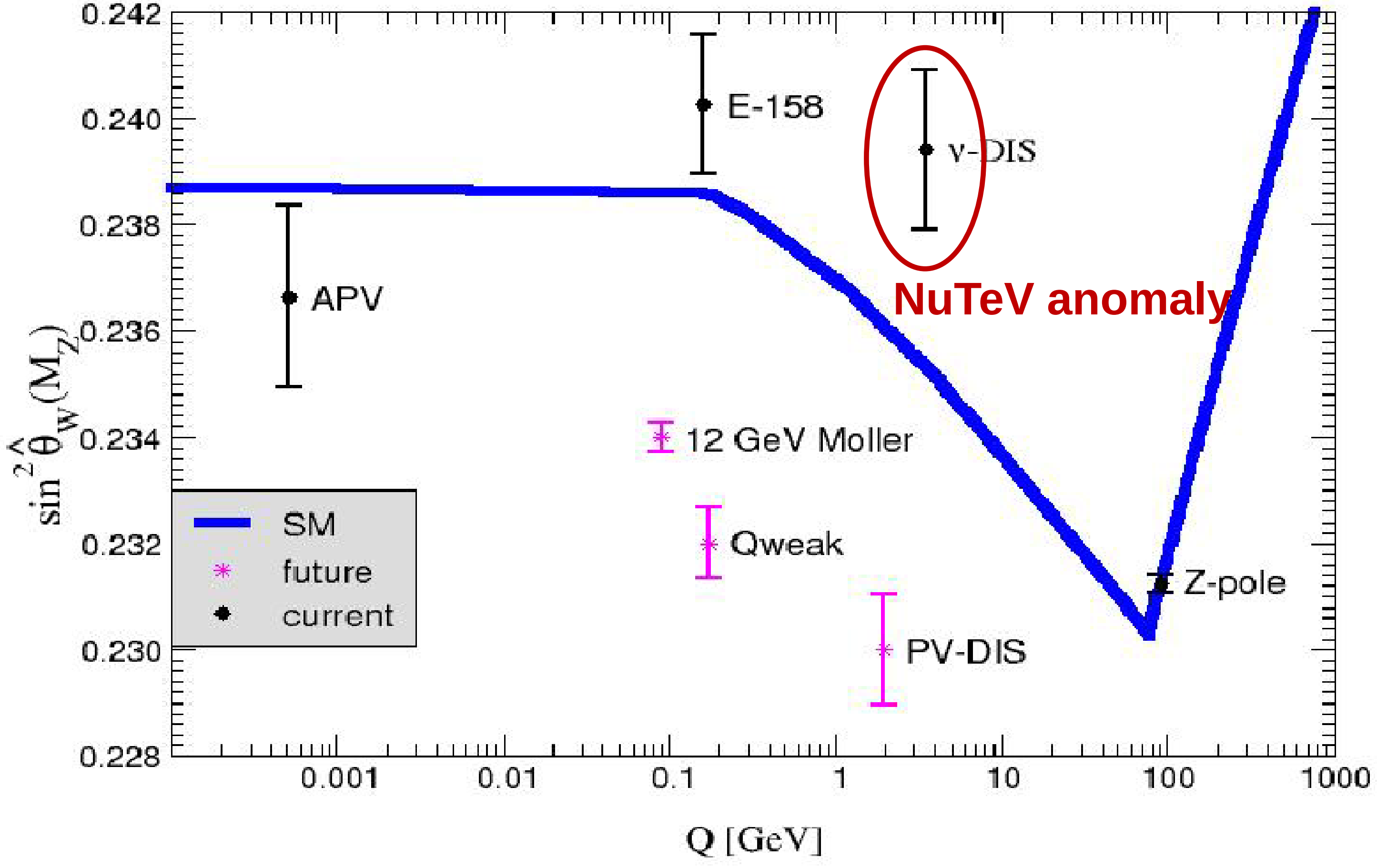


SUMMARY: This new correction is large but under control and with it Q_{weak} can achieve its goal

The NuTeV anomaly

Radiative Corrections as Standard Model Test

18 months ago….



NuTeV anomaly

Paschos-Wolfenstein Ratio

NuTeV measured (approximately) P-W ratio:

$$R^{PW} = \frac{\sigma(\nu\,\mathrm{Fe} \to \nu\,X) - \sigma(\bar{\nu}\,\mathrm{Fe} \to \bar{\nu}\,X)}{\sigma(\nu\,\mathrm{Fe} \to \mu^{-}\,X) - \sigma(\bar{\nu}\,\mathrm{Fe} \to \mu^{+}\,X)} = \frac{NC}{CC}\ \mathrm{ratio}$$

$$= \tfrac{1}{2} - \sin^2\theta_W$$

NuTeV

$\sin^2\theta_W = 1 - M_W^2/M_Z^2$ = 0.2277 ± 0.0013 ± 0.0009

other methods

c.f. Standard Model = 0.2227 ± 0.0004

(c.f. 1978: 0.230 ± 0.015)

NuTeV Anomaly

Phys. Rev. Lett. 88 (2002) 091802 : 409 citations since….

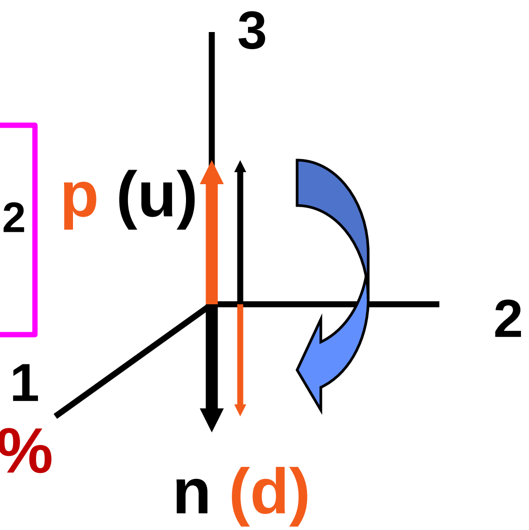
Fermilab press conference, Nov. 7, 2001:

“We looked at sin^2 θ_W ,” said Sam Zeller. The predicted value was 0.2227. The value we found was 0.2277…. might not sound like much, but the room full of physicists fell silent when we first revealed the result.”

“3 σ discrepancy) 99.75% probability ν are not like other particles…. only 1 in 400 chance that our measurement is consistent with prediction ,” MacFarland said.

Charge Symmetry

Its <u>assumed</u> that charge symmetry: $\boxed{e^{i\pi I_2}}$ is exact.



Good at < 1% : e.g. $(m_n - m_p) / m_p \sim 0.1\%$

That is: $u \equiv u^p = d^n$

$d \equiv d^p = u^n$ etc.

Hence:

$$F_2^{\,n} = 4/9 \times (d(x) + \bar{d}(x)) + 1/9 (u(x) + \bar{u}(x))$$

up-quark in n

down-quark in n

Summary of Charged Current Cross Section

$$\sigma_{CC}(\nu\ N{=}Z) \sim x\ \{\ (u + d + 2s) + \tfrac{1}{3}\ (\bar{u} + \bar{d} + 2\bar{c})\ \}$$

$$\sigma_{CC}(\bar{\nu}\ N{=}Z) \sim x\ \{\ \tfrac{1}{3}\ (u + d + 2c) + (\bar{u} + \bar{d} + 2\bar{s})\ \}$$

and hence:

$$\sigma_{CC}\ (\nu\ N{=}Z) - \sigma_{CC}\ (\bar{\nu}\ N{=}Z) = 2/3\ x\ \{u - \bar{u} + d - \bar{d}\} + 2\ x\ \{s - \bar{s}\} + 2/3\ x\ \{c - \bar{c}\}$$

$$= 2/3\ x\ (\ u_V + d_V\ ) + \ldots$$

(Valence distributions: $\int dx\ u_V = 2\ ;\ \int dx\ d_V = 1$ **)**

Neutral Current Cross Section

Z coupling	**g_L**	**g_R**
u, c, t	**$+1/2 - 2/3 \sin^2\theta_w$**	**$-2/3 \sin^2\theta_W$**
d, s, b	**$-1/2 + 1/3 \sin^2\theta_w$**	**$+1/3 \sin^2\theta_W$**

In Cross Section :

$\nu\, q_L \sim 1$; $\nu\, q_R \sim 1/3$

$\nu\, \bar{q}_L \sim 1/3$; $\nu\, \bar{q}_R \sim 1$

Hence, <u>for N=Z nucleus</u>: defining $g^2_L = g^2_{Lu} + g^2_{Ld} = ½ - \sin^2\theta_W + 5/9 \sin^4\theta_W$

and $g^2_R = g^2_{Ru} + g^2_{Rd} = 5/9 \sin^4\theta_W$

$$\sigma_{NC}(\nu A) \sim (g^2_L + g^2_R/3) \times (u + d) + (g^2_R + g^2_L/3) \times (\bar{u} + \bar{d})$$

$$\sigma_{NC}(\bar{\nu} A) \sim (g^2_L + g^2_R/3) \times (\bar{u} + \bar{d}) + (g^2_R + g^2_L/3) \times (u + d)$$

Finally : Paschos-Wolfenstein

$$\sigma_{NC}(\nu\,A) - \sigma_{NC}(\bar{\nu}\,A) \sim 2/3\,(\,g^2_L - g^2_R)\ x\ (u_V + d_V\,)$$

c.f. $\sigma_{CC}(\nu\,N{=}Z) - \sigma_{CC}(\bar{\nu}\,N{=}Z) \sim 2/3\ x\ (\,u_V + d_V\,)$

and therefore ratio of NC to CC cross section differences is

$$\boxed{R^{PW} = g^2{}_L - g^2{}_R = \tfrac{1}{2} - \sin^2\theta_W}$$

Provided: i) Charge Symmetry ii) $s(x) = \bar{s}(x)$

iii) $c(x) = \bar{c}(x)$ iv) No higher-twist effects (e.g. VMD shadowing)

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Correction to Paschos-Wolfenstein from CSV

- General form of the correction is:

$$\Delta R_{\mathrm{PW}} \simeq \left(1 - \frac{7}{3} s_W^2\right) \frac{\langle x_A\, u_A^- - x_A\, d_A^- - x_A\, s_A^- \rangle}{\langle x_A\, u_A^- + x_A\, d_A^- \rangle}$$

- $u_A = u^p + u^n$; $d_A = d^p + d^n$ and hence

  $u_A - d_A = (u^p - d^n) - (d^p - u^n) \equiv \delta u - \delta d$

- N.B. In general the corrections are C-odd and so involve only valence distributions: $q^- = q - \bar{q}$

Davidson *et al*., hep-ph/0112302

Estimates of Charge Symmetry Violation*

- Origin of effect is $m_d \neq m_u$
- Unambiguously predicted : $\boxed{\delta d_V - \delta u_V > 0}$
- Biggest % effect is for minority quarks, i.e. δd_V
- Same physics that gives : d_v / u_V small as $x \to 1$

  and : g^p_1 and $g^n_1 > 0$ at large x

Close & Thomas, Phys Lett B212 (1988) 227

i.e. mass difference of quark pair spectators to hard scattering

---

*** Sather, Phys Lett B274 (1992) 433;**
Rodionov et al., Mod Phys Lett A9 (1994) 1799

Non-Perturbative Structure of Nucleon

To calculate PDFs need to evaluate non-perturbative matrix elements

Using either : i) lattice QCD or ii) Model

i) Lattice QCD can only calculate low moments of $u^p - d^p$

quite a lot has been learnt....

BUT nothing yet about CSV

ii) Model uses same methods that successfully explain d/u at large x, dominance of $u\uparrow$ at large x, etc…

(Close & Thomas: 1988)

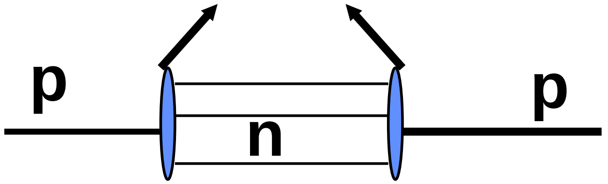
Di-quark Spectator States Dominate Valence

For s-wave valence quarks, most likely three-momentum is zero :

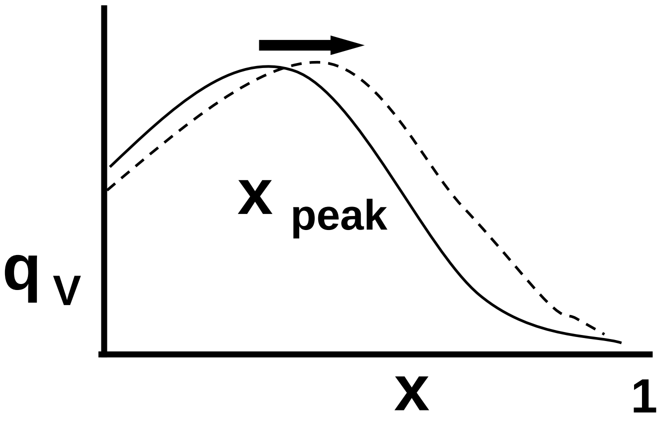
$\delta(M\,(1 - x) - m_n)$ determines x where $q\,(x, Q^2_0)$ is maximum

i.e. $\boxed{x_{peak} = (M - m_n) / M}$ and hence lowest $m_n \rightarrow$ large – x behaviour

Natural choice is two-quark state



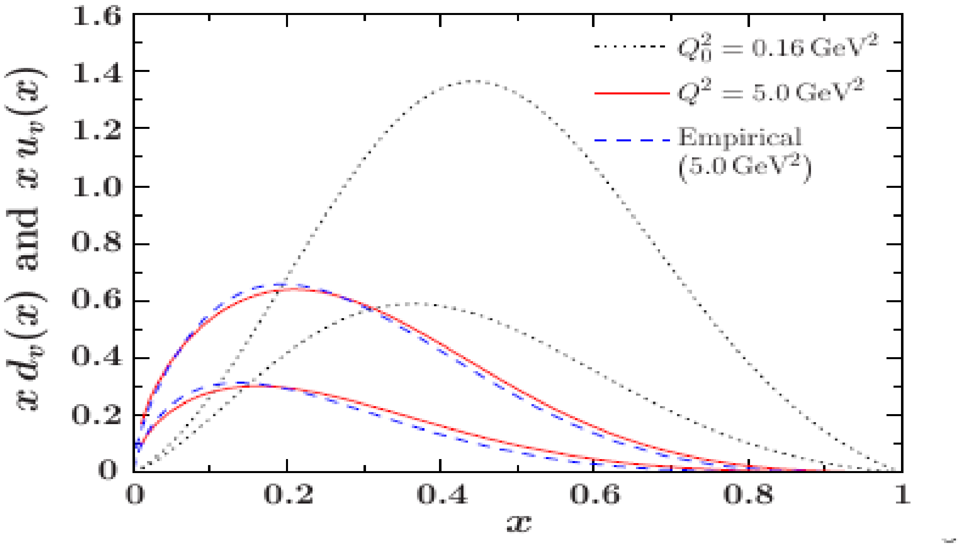
$m_2 / M = 2/3$ (CQM);
$= 3/4$ MIT bag ➡ $x_{peak} \sim 1/4$ to $1/3$

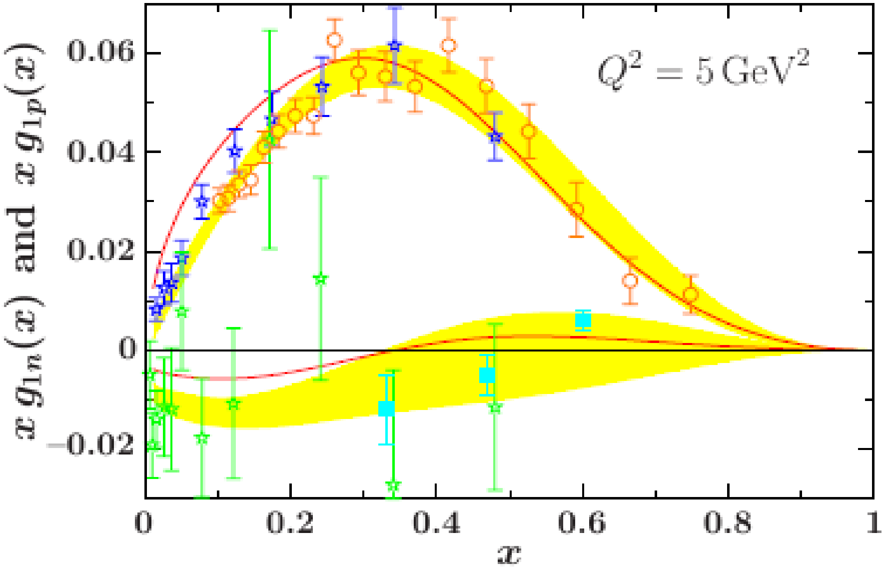


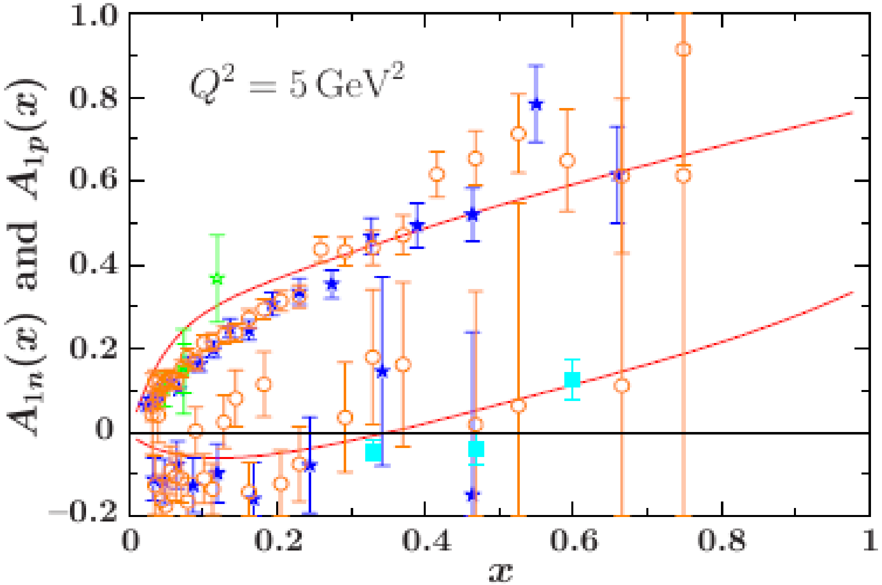
If $m_2 \downarrow$: x_{peak} moves to right

More Modern (Confining) NJL Calculations

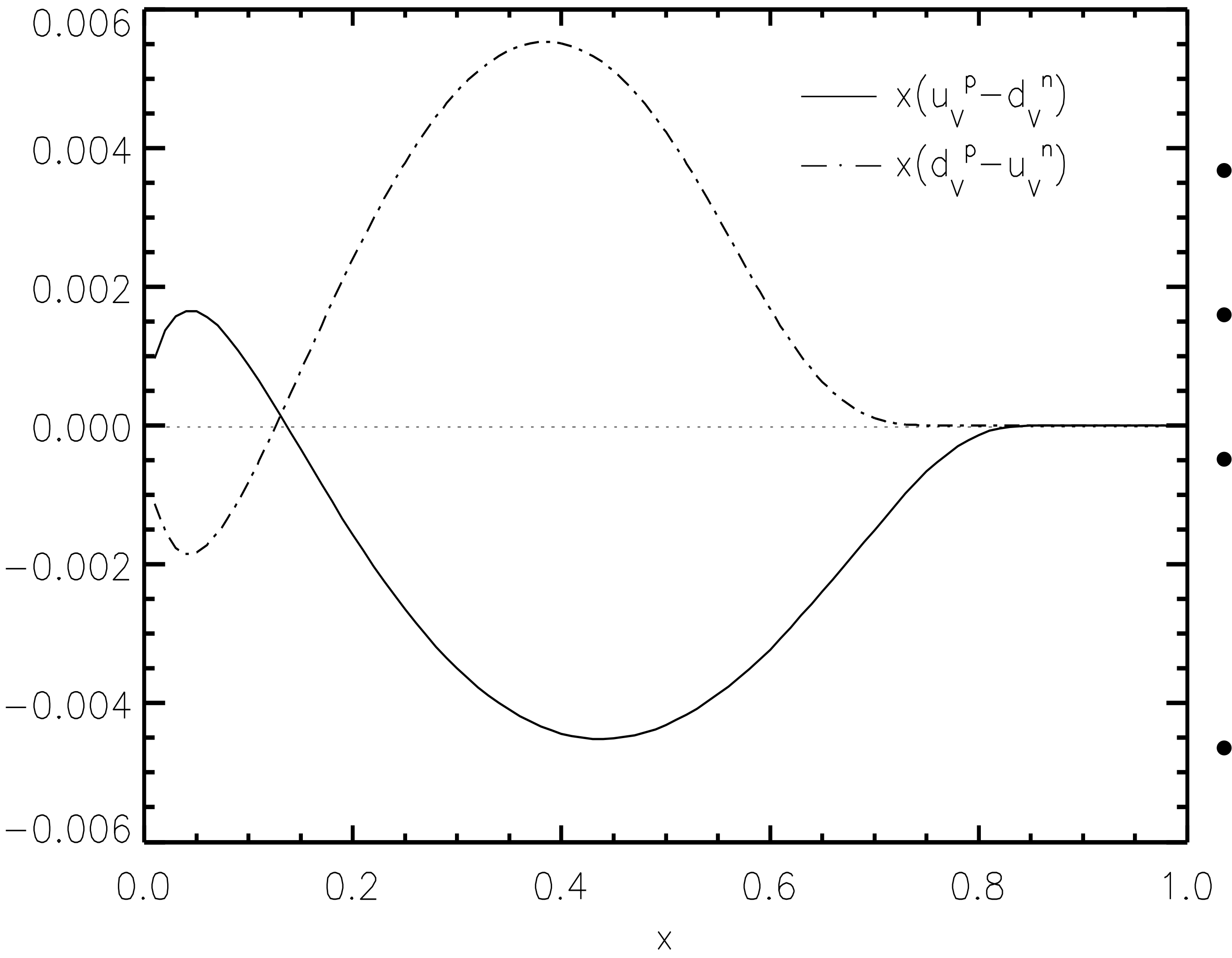
Cloet et al.,
Phys. Lett. B621, 246 (2005)
(μ = 0.4 GeV)







Application to Charge Symmetry Violation



- d in p : uu left
- u in n : dd left
- Hence m_2 lower by about 4 MeV for d in p than u in n
- Hence $d^p > u^p$ at large x.

From: Rodionov et al., Mod Phys Lett A9 (1994) 1799

Remarkably Similar to Recent MRST Fit

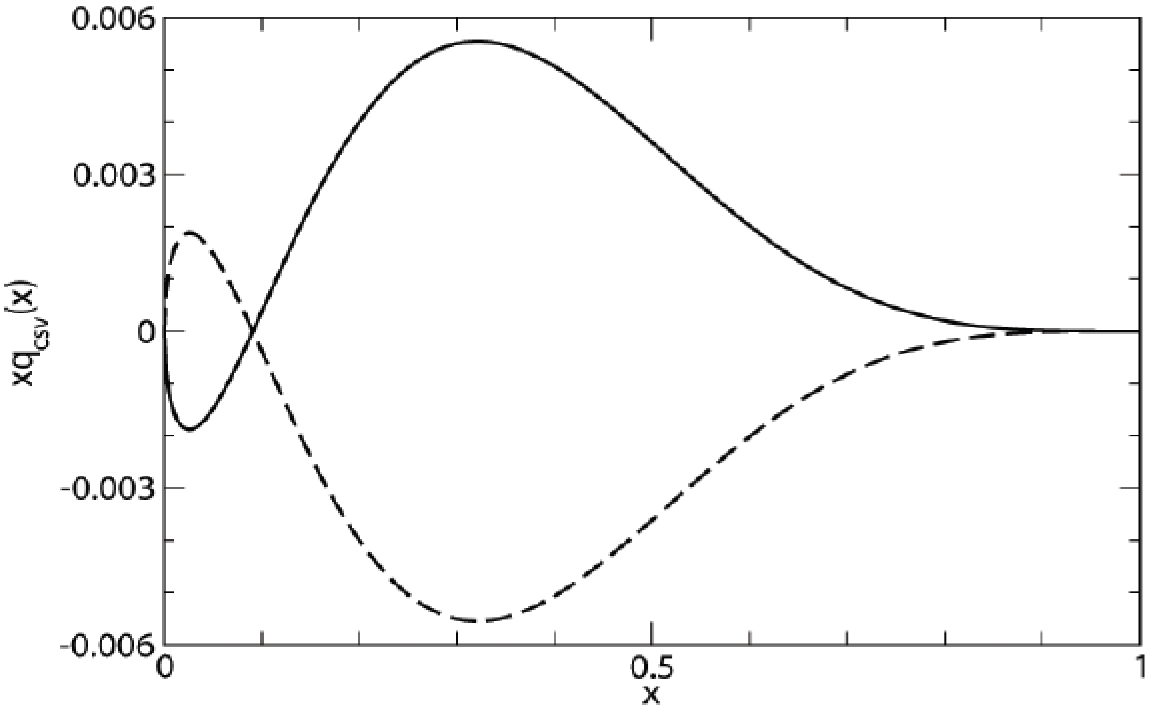


FIG. 5: The phenomenological valence quark CSV function from Ref. [23], corresponding to best fit value $\kappa = -0.2$ defined in Eq. (35). Solid curve: $x\delta d_{\rm v}$; dashed curve: $x\delta u_{\rm v}$.

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Model Calculations Reduce NuTeV by 1σ

Two original ('92 and '93) calculations agree very (too?) well with each other and with recent approximation based on phenomenological PDFs

Includes effect of NuTeV acceptance

(Zeller *et al*., hep-ex/0203004)

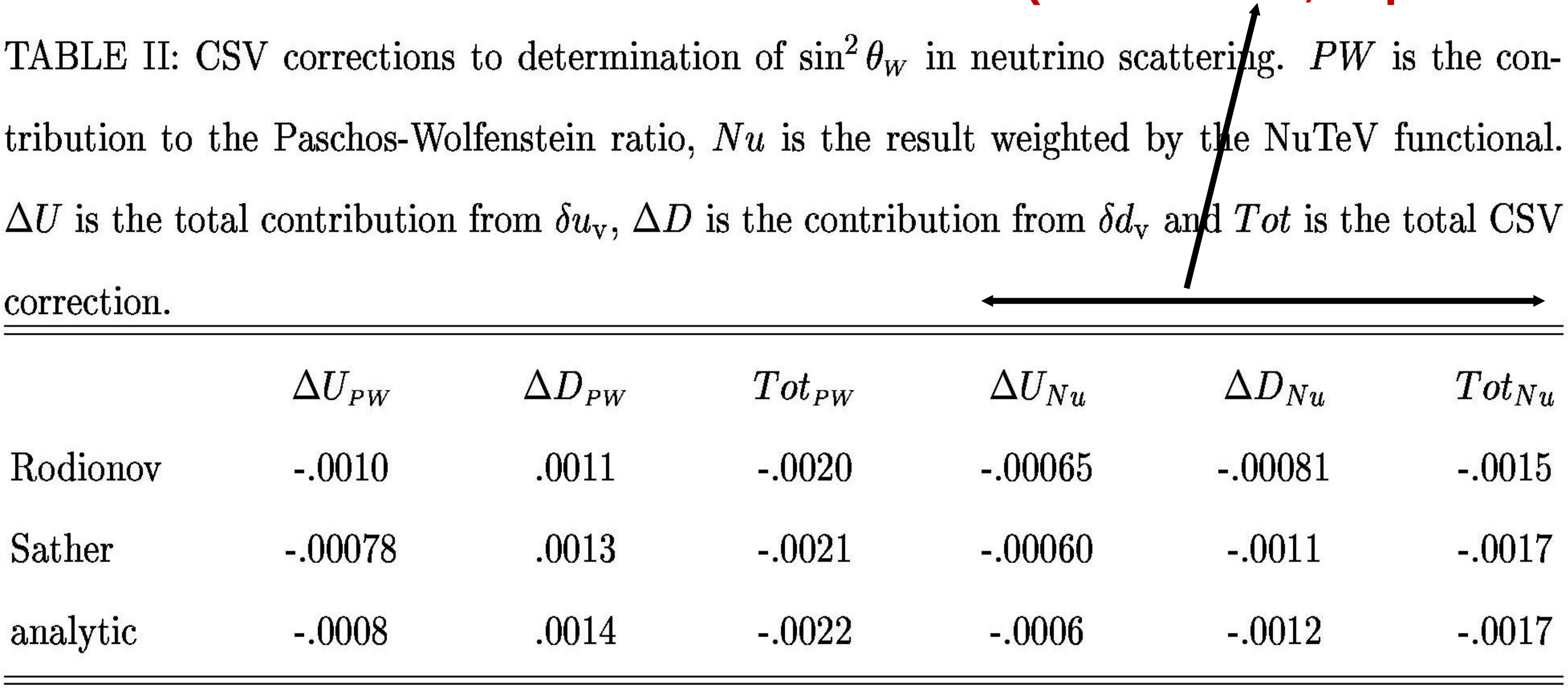


TABLE II: CSV corrections to determination of $\sin^2\theta_W$ in neutrino scattering. *PW* is the contribution to the Paschos-Wolfenstein ratio, *Nu* is the result weighted by the NuTeV functional. ΔU is the total contribution from $\delta u_{\rm v}$, ΔD is the contribution from $\delta d_{\rm v}$ and *Tot* is the total CSV correction.

	ΔU_{PW}	ΔD_{PW}	Tot_{PW}	ΔU_{Nu}	ΔD_{Nu}	Tot_{Nu}
Rodionov	-.0010	.0011	-.0020	-.00065	-.00081	-.0015
Sather	-.00078	.0013	-.0021	-.00060	-.0011	-.0017
analytic	-.0008	.0014	-.0022	-.0006	-.0012	-.0017

Londergan & Thomas, Phys Lett B558 (2003) 132

Indeed : Can Show Very Nearly Model Independent*

$$\delta D_V = \frac{\delta M}{M} D_V + \frac{\delta m_2}{M} \sim 0.0046$$

$$\delta U_V = \frac{\delta M}{M}(U_V - 2) \sim -0.0020$$

Small dependence on "bag / quark model" scale (Q^2_0) :

$D_V \sim 0.2$: $U_V \sim 0.6$ − i.e. 10% & 30% respectively

Correction to Paschos-Wolfenstein is therefore :

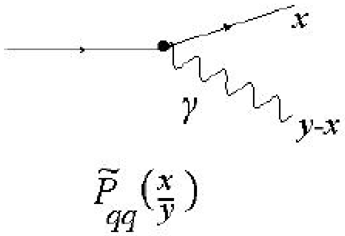
$$\Delta R^{PW} = 0.5\,(g^2_L - g^2_R)\,\frac{\delta U_V - \delta D_V}{U_V + D_V} \sim -0.0020$$

N.B. Ratio of non-singlet moments independent of Q^2 under NLO evolution

Londergan and Thomas, PR D67 (2003) 111901

An additional source of CSV

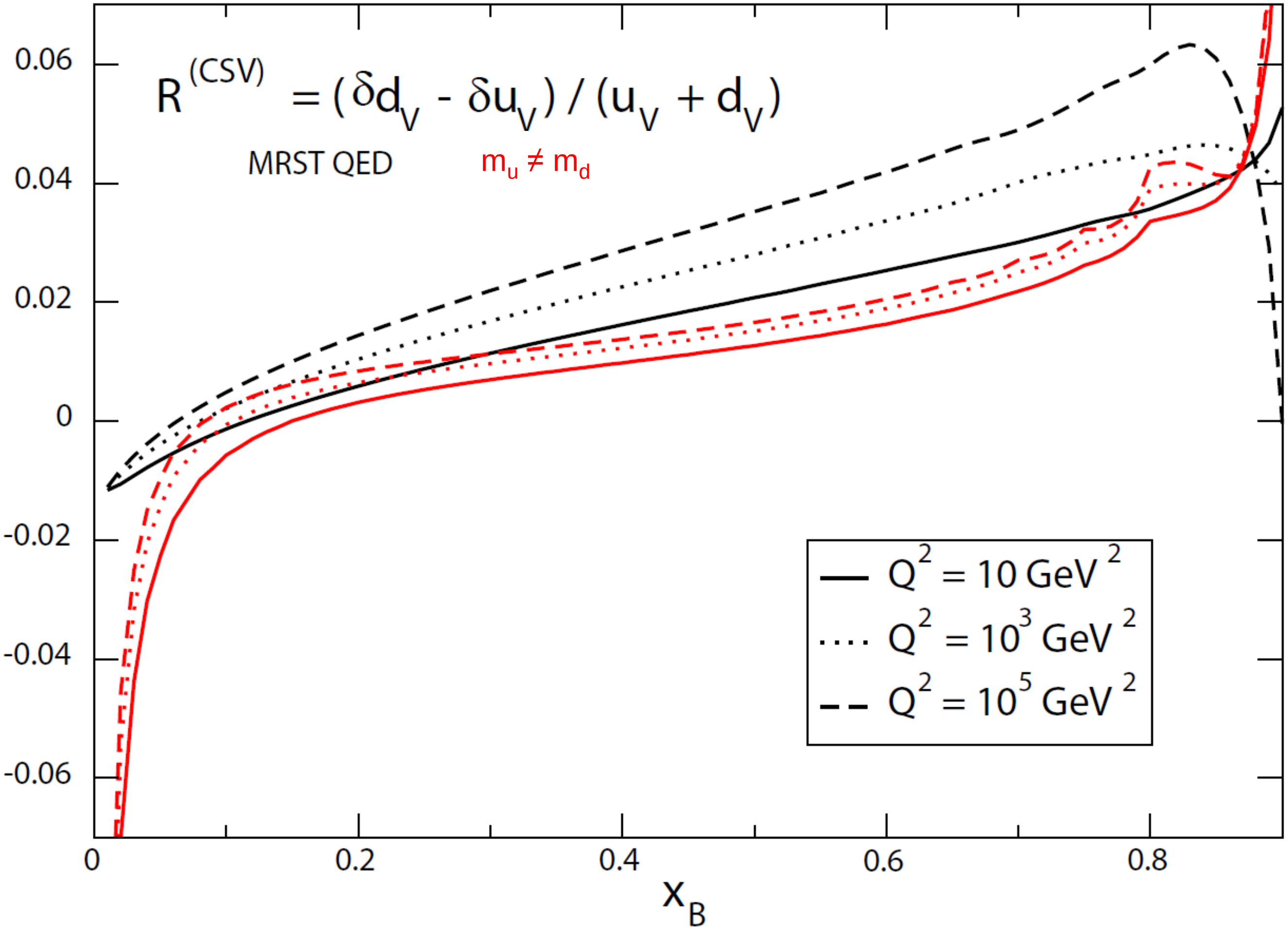
- In addition to the u-d mass difference, MRST (Eur Phys J C39 (2005) 155) and Glück et al (PRL 95 (2005) 022002) suggested that "QED splitting":



- which is obviously larger for u than d quarks, would be an additional source of CSV. Assume zero at some low scale and then evolve − so CSV from this source grows with Q^2
- Effect on NuTeV is exactly as for regular CSV and magnitude but grows logarithmically with Q^2
- For NuTeV it gives: $\Delta R^{\mathrm{QED}} = -0.0011$ to which we assign 100% error

EIC an Ideal Place to test QED Splitting

- **Effect increases with Q^2. Use (e^-, ν) and (e^+, $\bar{\nu}$) on p and d**
- **This gives CSV and d/u unambiguously**



Hobbs, Londergan and Thomas, in preparation

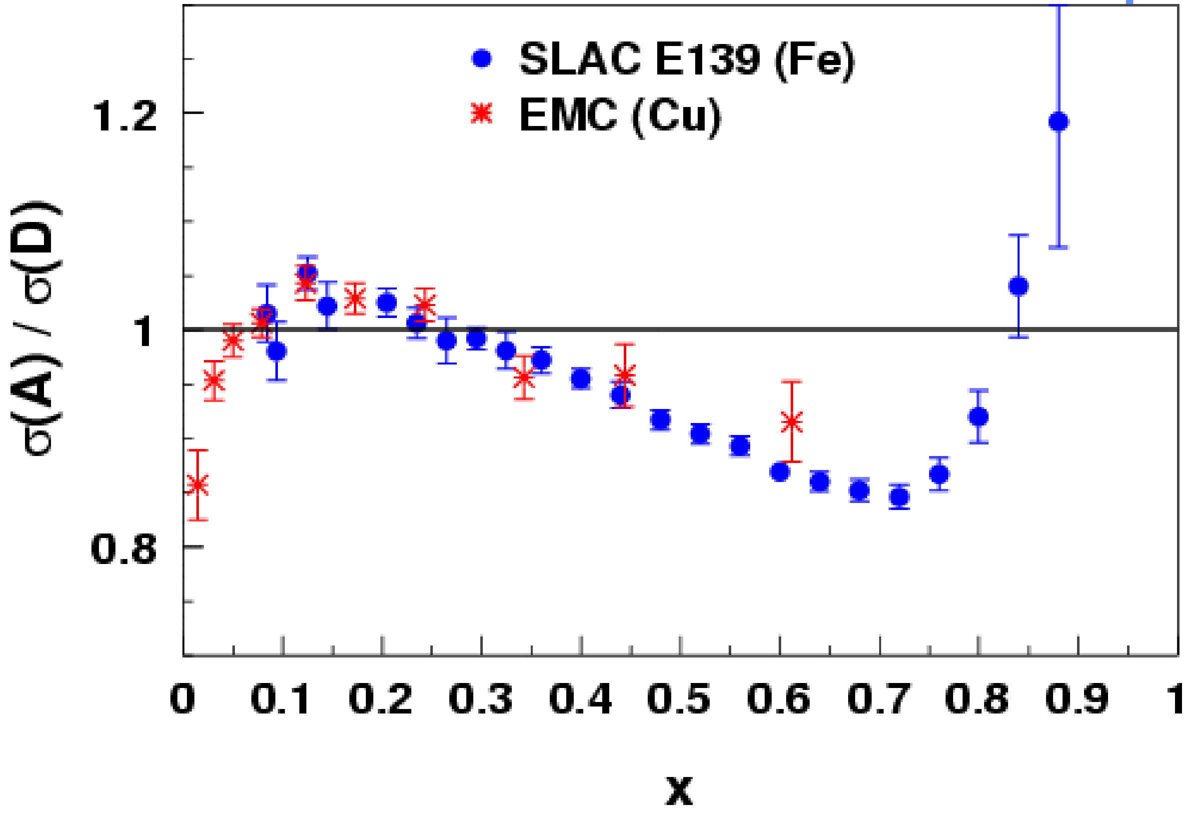
Isovector EMC Effect

The EMC Effect: Nuclear PDFs

- Observation **stunned and electrified** the
  HEP and Nuclear communities 20 years ago

- Nearly 1,000 papers have been generated…..

- Medium modifies the momentum distribution of the quarks!



J. Ashman *et al.*, Z. Phys. C57, 211 (1993)

J. Gomez *et al.*, Phys. Rev. D49, 4348 (1994)

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Recent Calculations for Finite Nuclei

Spin dependent EMC effect TWICE as large as unpolarized

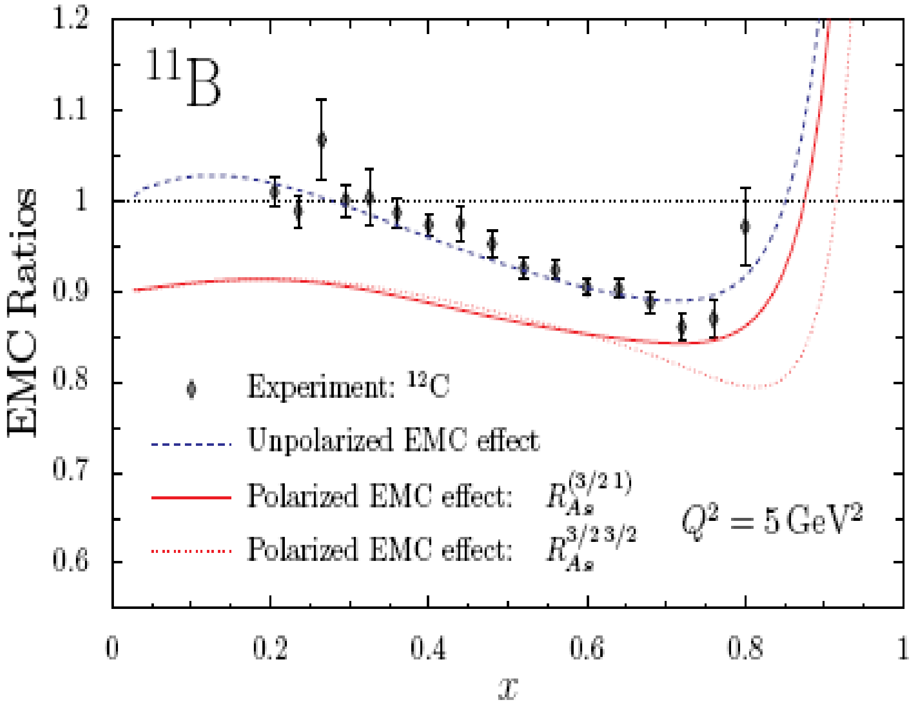


FIG. 7: The EMC and polarized EMC effect in ^{11}B. The empirical data is from Ref. [31].

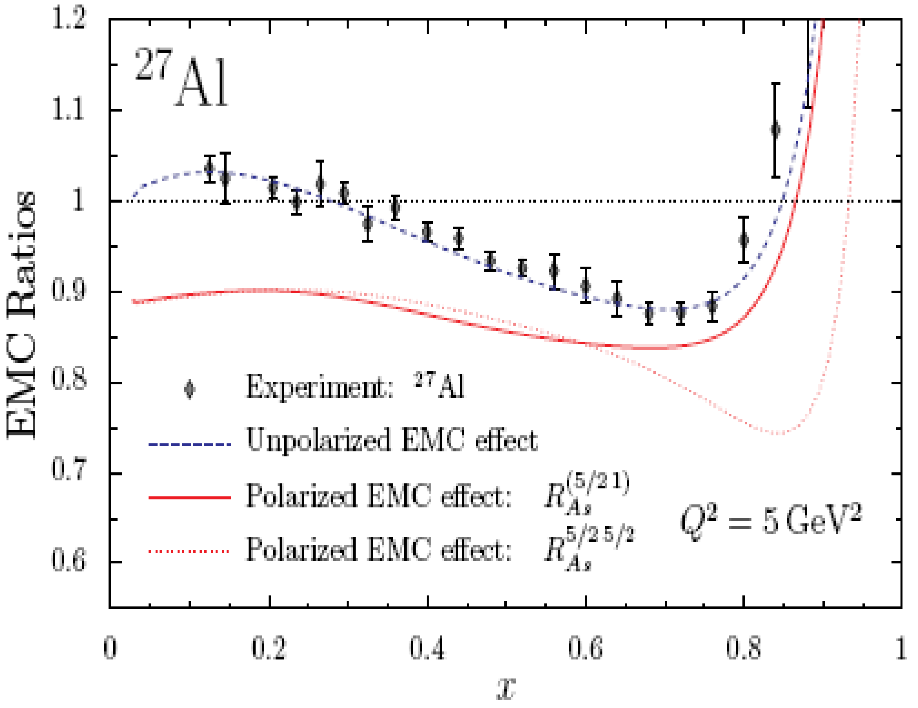


FIG. 9: The EMC and polarized EMC effect in ^{27}Al. The empirical data is from Ref. [31].

Cloet, Bentz, Thomas, Phys. Lett. B642 (2006) 210 (nucl-th/0605061)

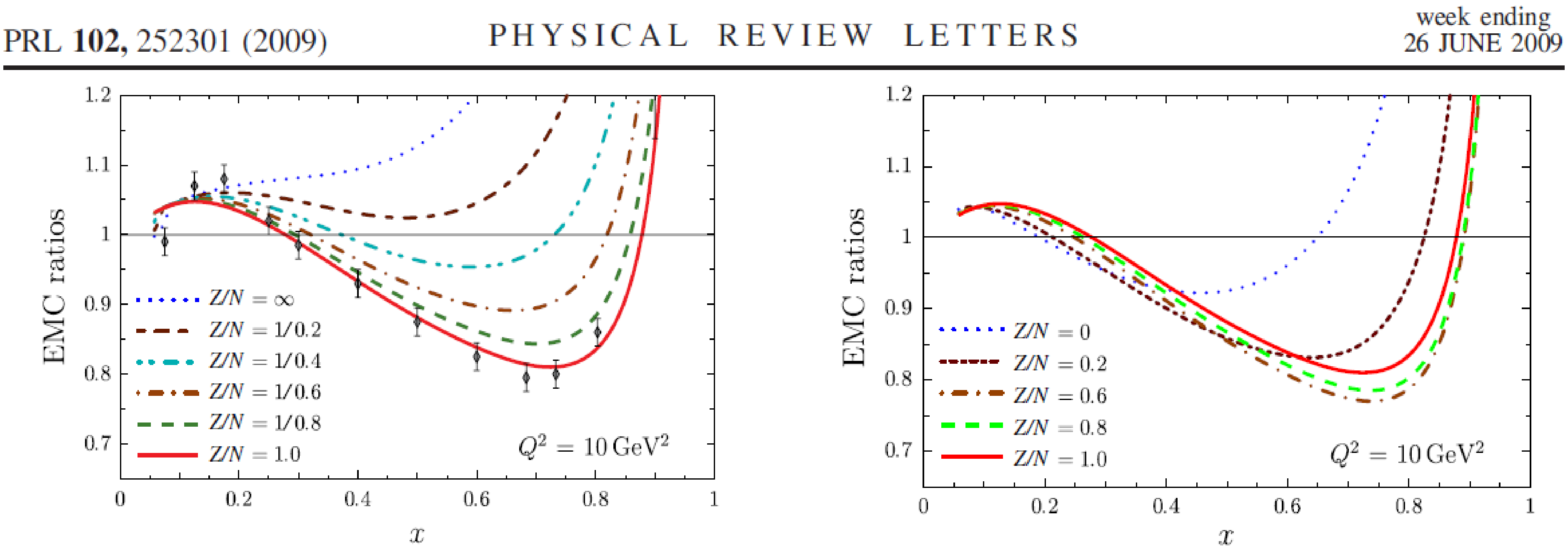
Recently Discovered Iso-vector EMC Effect

- **New realization concerning EMC effect:**
  - **isovector force in nucleus (like Fe) with N≠Z effects ALL u and d quarks in the nucleus**
  - **subtracting structure functions of extra neutrons is not enough**
  - ***there is a shift of momentum from all u to all d quarks***
- **This has same sign as charge symmetry violation associated with $m_u \neq m_d$**
- **Sign and magnitude of both effects exhibit little model dependence**

Cloet et al., arXiv: 0901.3559v1 ;
Londergan et al., Phys Rev D67 (2003) 111901

Isovector EMC Effect

Cloet, Bentz, Thomas



PRL **102,** 252301 (2009) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2009

$$q(x) = \frac{p^+}{p^+ - V^+}\, q_0\!\left(\frac{p^+}{p^+ - V^+}\, x - \frac{V_q^+}{p^+ - V^+}\right)$$

Correction to Paschos-Wolfenstein from ρ_p - ρ_n

$$\Delta R_{\mathrm{PW}} \simeq \left(1 - \frac{7}{3} s_W^2\right) \frac{\langle x_A\, u_A^- - x_A\, d_A^- - x_A\, s_A^- \rangle}{\langle x_A\, u_A^- + x_A\, d_A^- \rangle}$$

- **Excess of neutrons means d-quarks feel more repulsion than u-quarks**

- **Hence shift of momentum from all u to all d in the nucleus!**

- **Negative change in ΔR_{PW} and hence $\sin^2\theta_W$ ↑**

- **Isovector force controlled by $\rho_p - \rho_n$ and symmetry energy of nuclear matter − both well known!**

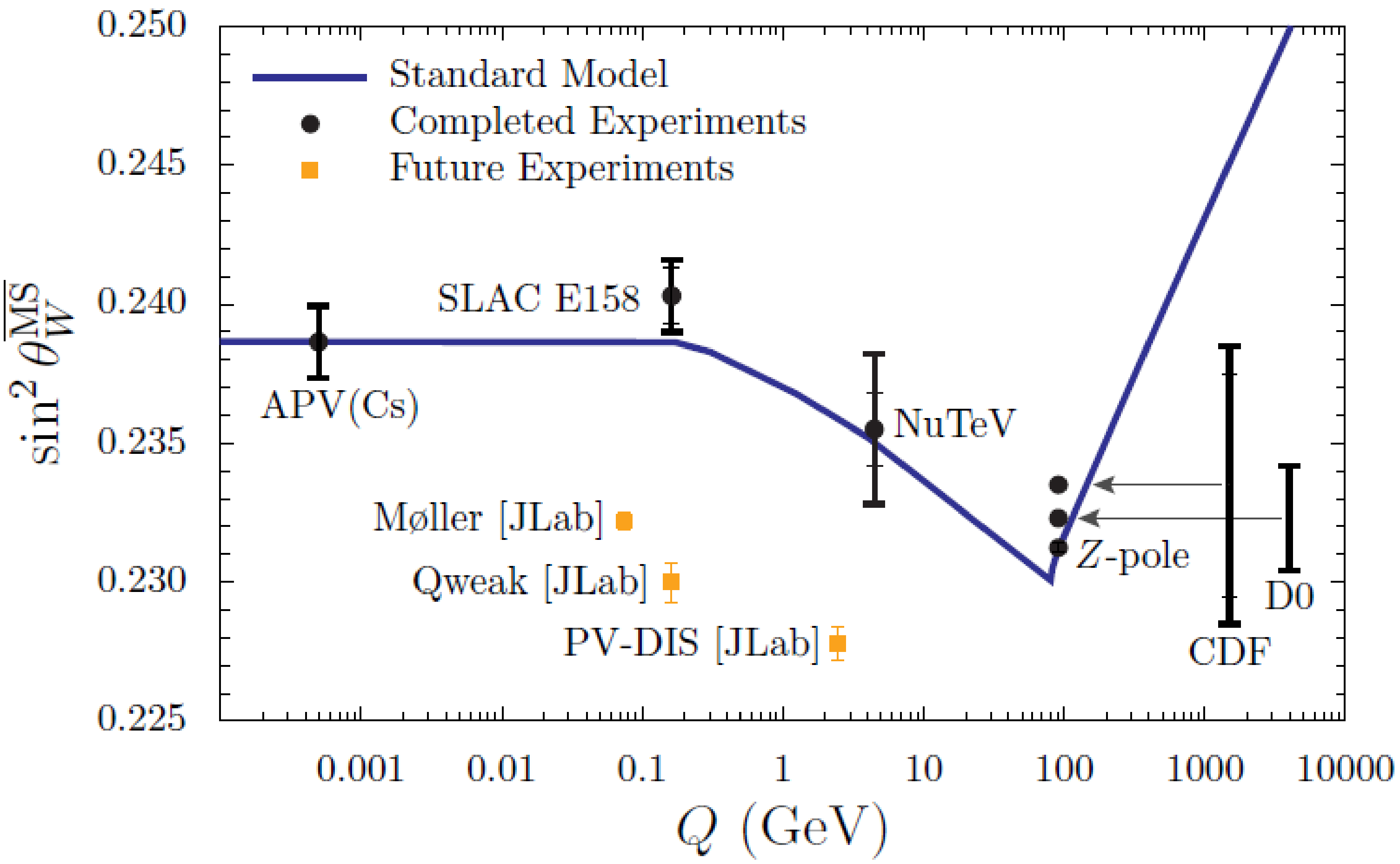
- **N.B. ρ^0 mean field included in QHD and QMC and earlier work with Bentz but no-one thought of this!!**

Summary of Corrections to NuTeV Analysis

- **Isovector EMC effect:** $\Delta R^{\rho^0} = -0.0019 \pm 0.0006$
  - **using NuTeV functional**
- **CSV:** $\Delta R^{\mathrm{CSV}} = -0.0026 \pm 0.0011$
  - **again using NuTeV functional**
- **Strangeness:** $\Delta R^{s} = 0.0 \pm 0.0018$
  - **this is largest uncertainty (systematic error) ; desperate need for an accurate determination of $s^-(x)$, e.g. semi-inclusive DIS?**
- **Final result:** $\sin^2\theta_W = 0.2232 \pm 0.0013(\mathrm{stat}) \pm 0.0024(\mathrm{syst})$
  - **c.f. Standard Model:** $\sin^2\theta_W = 0.2227 \pm 0.0004$

The Standard Model Works Again

Apply CSV and isovector EMC corrections
plus estimate systematic error arising from $s^-(x) \neq 0$:



Bentz et al., arXiv: 0908.3198

Separate Neutrino and Anti-neutrino Ratios

- **Biggest criticism of this explanation has been that NuTeV actually measured R^{ν} and $R^{\bar{\nu}}$, separately:**
  Claim we should compare directly with these.

- **Have done this:**

$$\delta R^{\nu} = \frac{2\,(3\,g_{Lu}^2 + g_{Ru}^2)\,\langle x_A\,u_A^- - x_A\,d_A^-\rangle}{\langle 3\,x_A\,u_A + 3\,x_A\,d_A + x_A\,\bar{u}_A + x_A\,\bar{d}_A + 6\,x_A\,s_A\rangle}$$

$$\delta R^{\bar{\nu}} = \frac{-2\,(3\,g_{Rd}^2 + g_{Ld}^2)\,\langle x_A\,u_A^- - x_A\,d_A^-\rangle}{\langle x_A\,u_A + x_A\,d_A + 3\,x_A\,\bar{u}_A + 3\,x_A\,\bar{d}_A + 6\,x_A\,\bar{s}_A\rangle}$$

- **Then R^{ν} moves from 0.3916 ± 0.0013 c.f. 0.3950 in the Standard Model to 0.3933 ± 0.0015;**

  $R^{\bar{\nu}}$ moves from 0.4050 ± 0.0027 to 0.4034 ± 0.0028, c.f. 0.4066 in SM

- **This is tremendous improvement :**
  chisq changes from 7.2 to 2.6 for the two ratios!

Bentz et al., arXiv: 0908.3198

Microscopic Derivation of Fragmentation Functions

- **Many critical problems in our field need to detect mesons in final state, in coincidence with one or more other particles**

- **TMDs this morning BUT also semi-inclusive DIS for flavor structure (e.g. $s^-(x)$, CSV , d/u...)**

- **Much of the work is extremely phenomenological**
  - **often guess functional forms, e.g. ratios of unfavoured to favoured fragmentation functions**

- **Want to draw attention to recent progress in the microscopic calculation of these functions using NJL model - by Matevosyan, Bentz, Cloet, Ito, Yazaki and Thomas**

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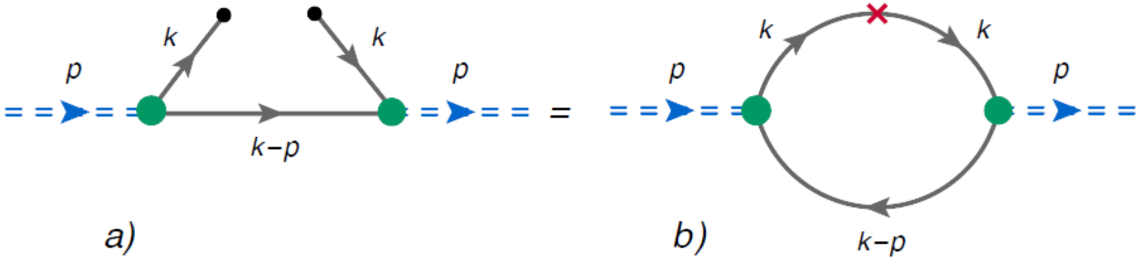
Microscopic Fragmentation Functions

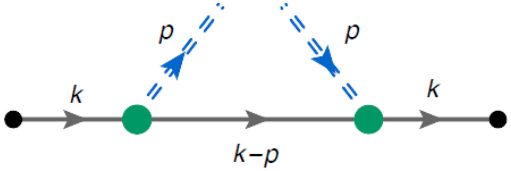
Elementary FFs and PDFs Related

- **Parton distribution functions:**



$$f_q^m(x) = iN_c \frac{C_I}{2} g_{mqq}^2 \int \frac{dk_+ d^2k_\perp}{(2\pi)^4} Tr[\gamma_5 S_1(k)\gamma_+ S_1(k)\gamma_5 S_2(k-p)]$$

- **Elementary fragmentation functions:**

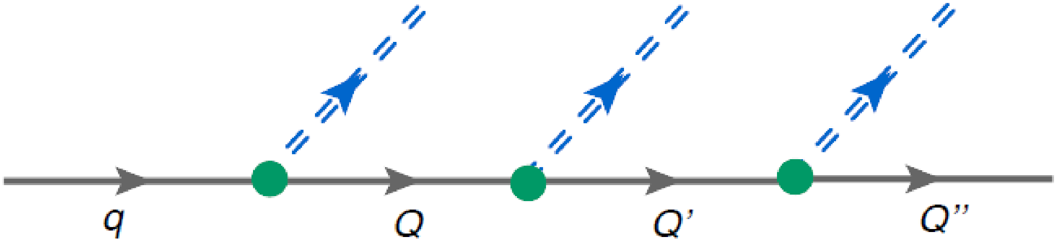


$$d_q^m(z) = N_c \frac{C_I}{2} g_{mqq}^2 \frac{z}{2} \int \frac{d^4k}{(2\pi)^4} Tr[S_1(k)\gamma_+ S_1(k)\gamma_5(\not{k} - \not{p} + M_2)\gamma_5]$$
$$\times \delta(k_- - p_-/z)\delta((p-k)^2 - M_2^2) = \frac{z}{2N_c} f_q^m(x = 1/z)$$

Ito et al., Phys Rev D80 (2009) 074008

NJL Jet Model

- In a semi-inclusive measurement see one pion BUT any number may have been emitted: must sum over the lot!

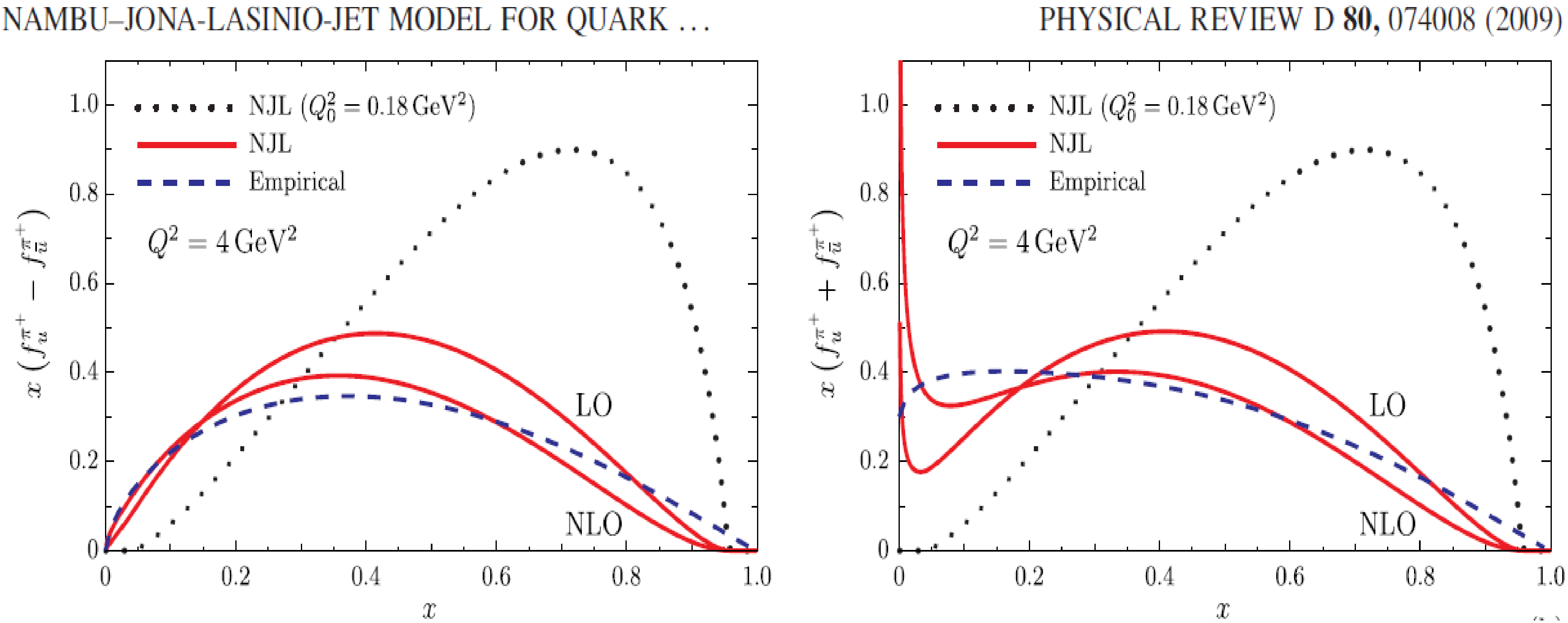


- Not doing so is the reason previous treatments impose overall normalization to match data

- We sum over all possibilities using coupled integral equations:

$$D_q^m(z) = \hat{d}_q^m(z) + \sum_Q \int_z^1 \frac{dy}{y} \hat{d}_q^Q\left(\frac{z}{y}\right) D_Q^m(y), \qquad \hat{d}_q^Q(z) = \hat{d}_q^m(1-z)\big|_{m=q\bar{Q}}$$

Pion PDFs in Bjorken Limit



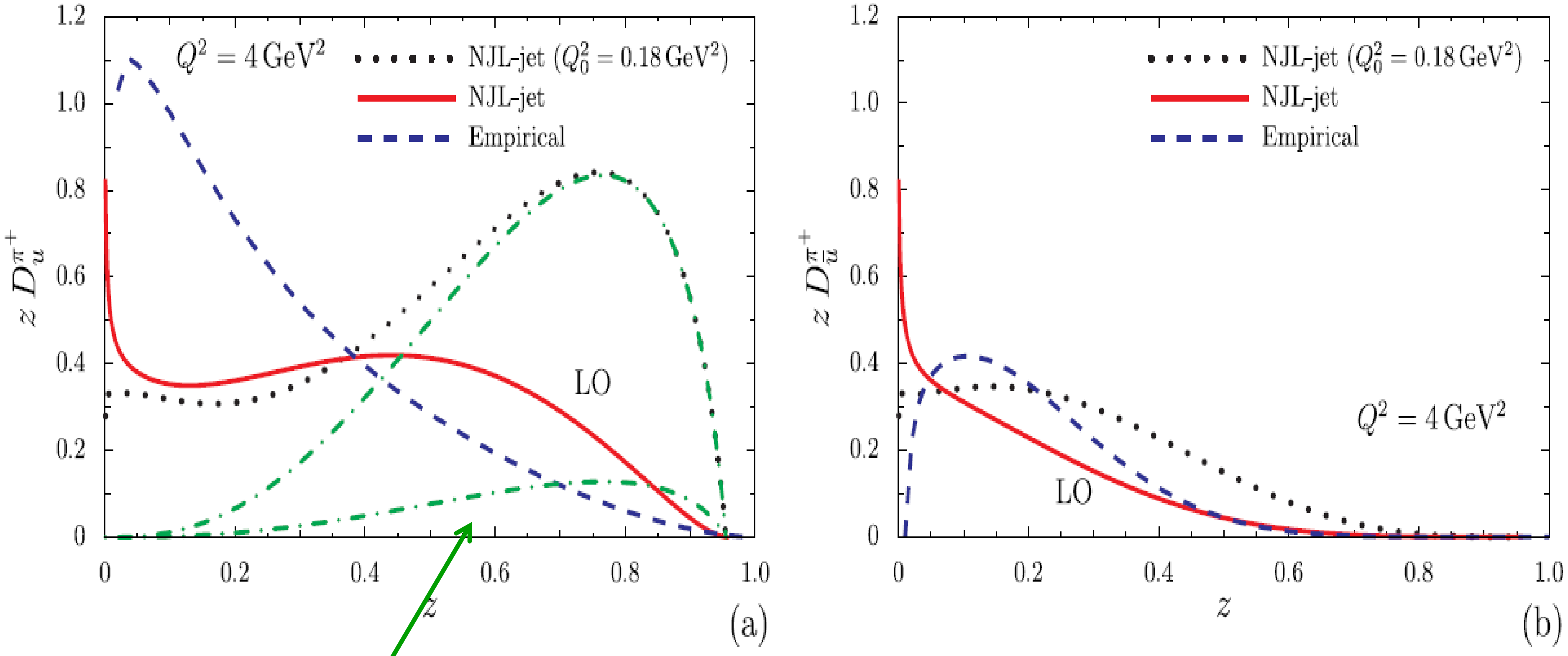
NAMBU–JONA-LASINIO-JET MODEL FOR QUARK …

PHYSICAL REVIEW D **80,** 074008 (2009)

Empirical: Sutton et al., Phys Rev D45 (1992) 2349

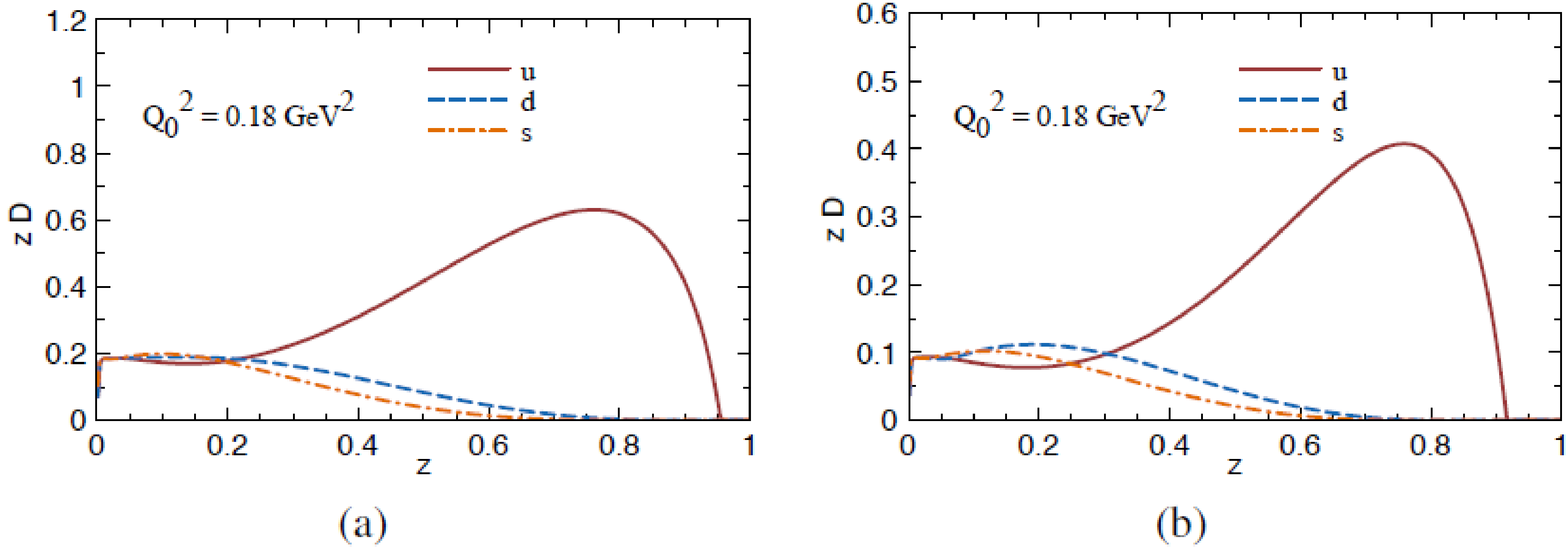
Corresponding Fragmentation Functions

ITO, BENTZ, CLOËT, THOMAS, AND YAZAKI PHYSICAL REVIEW D **80,** 074008 (2009)



Elementary fragmentation function

Add Strange Quark and Couple to K Fragmentation

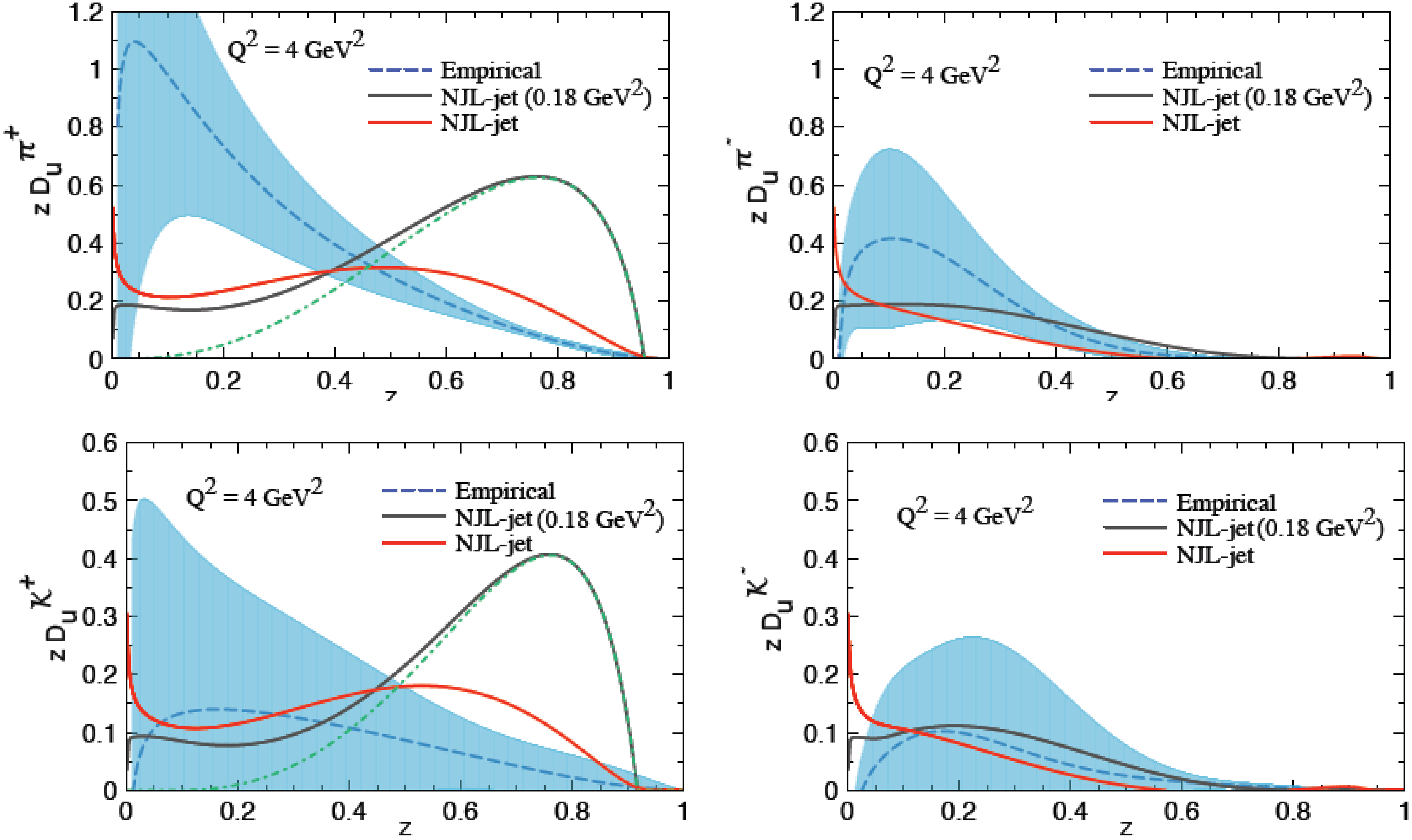


(a) (b)

FIGURE 4. a) π^+ and b) K^+ fragmentation functions at model scale $Q_0^2 = 0.18$ GeV^2.

Matevosyan et al., arXiv:1004.3075 [nucl-th]

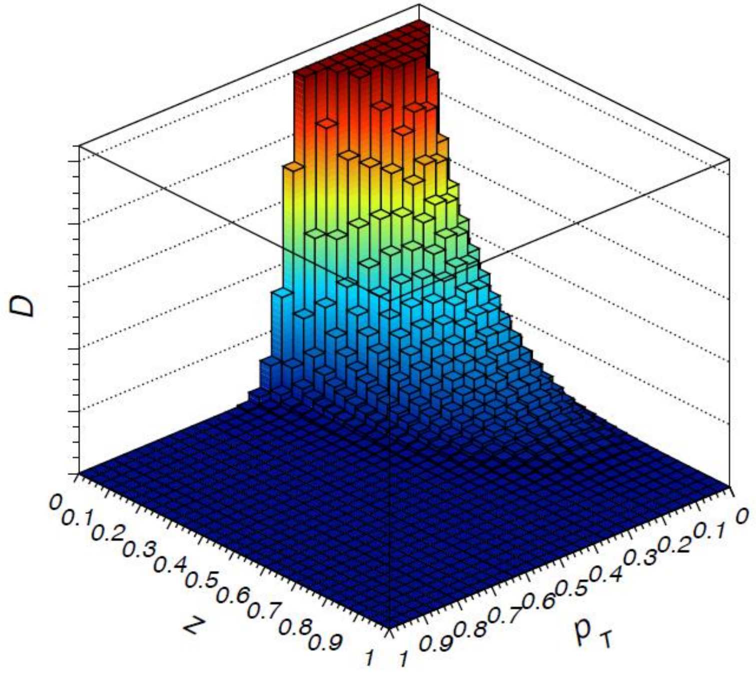
After Evolution – π and K FFs



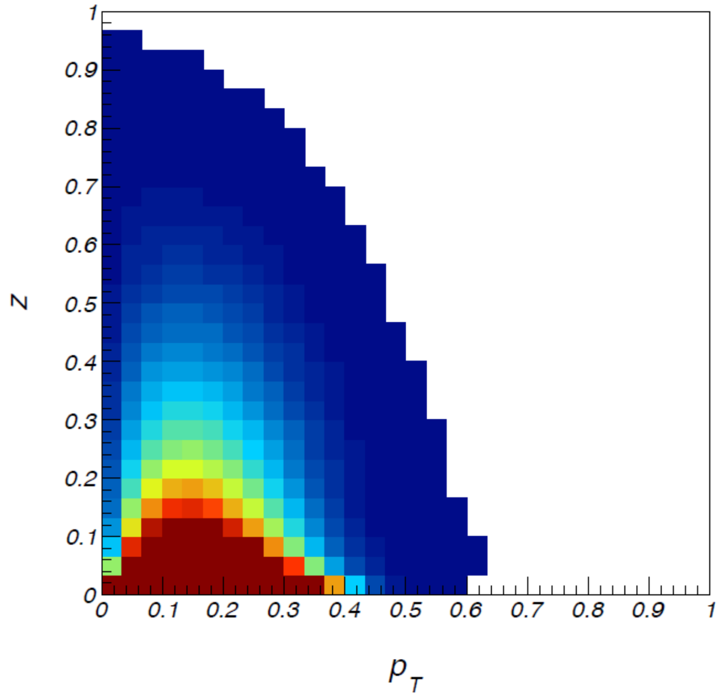
Matevosyan et al., arXiv:1004.3075 [nucl-th]

Now Turning to Finite Energy & non-Integrated Transverse Momentum – Monte Carlo Studies

$D_u^{\pi-}$



$D_u^{\pi-}$



Welcome interaction with experimentalists working on these problems. Monte Carlo methods allow us to match to experimental conditions – not just Bj limit

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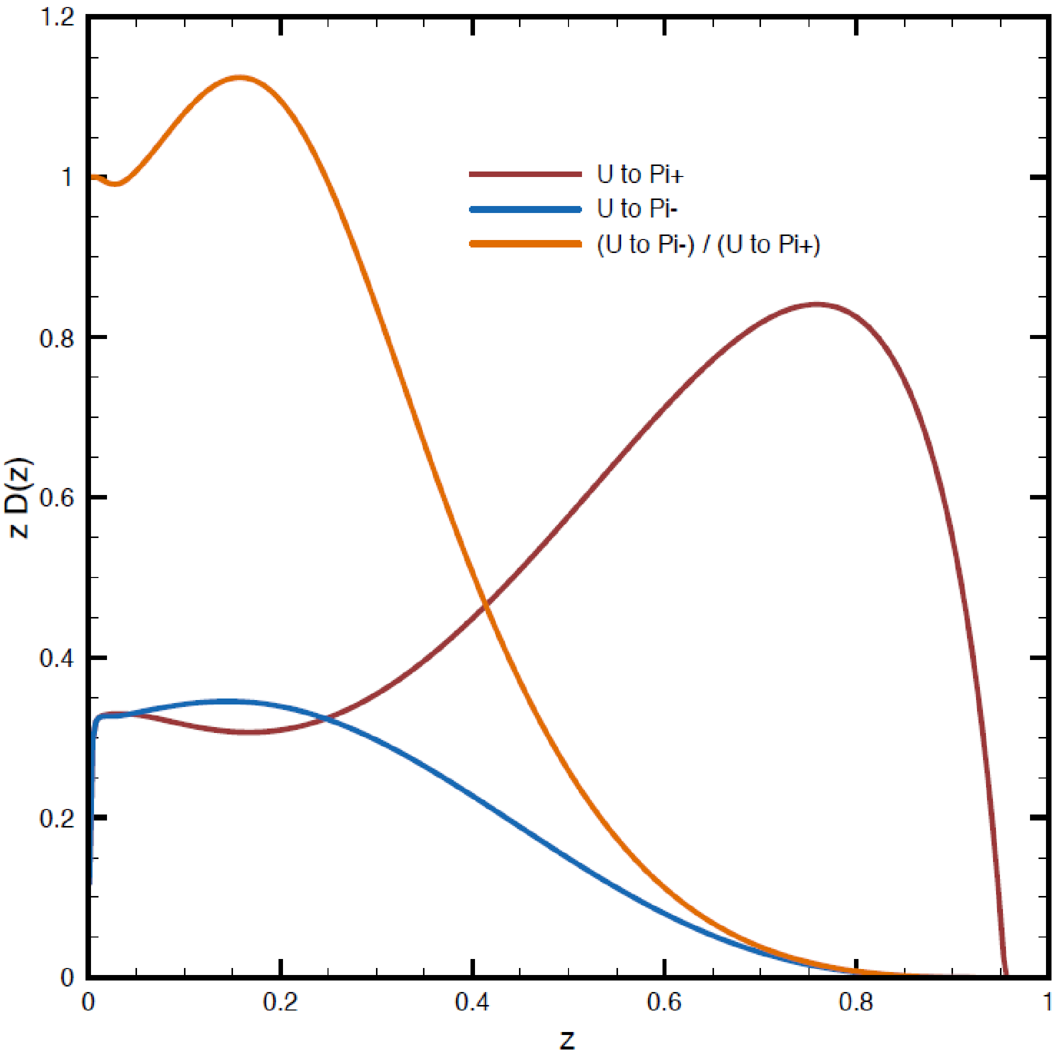
Ratio: Unfavoured to Favoured FFs



Matevosyan et al., in preparation

Summary

- JLab has made extremely important tests of fundamental features of the Standard Model
    - strange quarks as analog of Lamb shift in QED
    - weak charge of the proton

- In near future Q_{weak} has potential for further major advance

- The major outstanding discrepancy with Standard Model predictions for Z^0 was the NuTeV anomaly
    - this is resolved by CSV and newly discovered "isovector EMC effect"

- Can test these effects using CC reactions or parity violating DIS at an EIC

- Major remaining uncertainty is $s(x) - \bar{s}(x)$ …..

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Summary (cont.)

- Microscopic studies of fragmentation functions, including transverse momentum dependence showing promise
- Look forward to working with experimentalists to improve analysis of key data